THOMAS SIMPSON AND HIS TIMES

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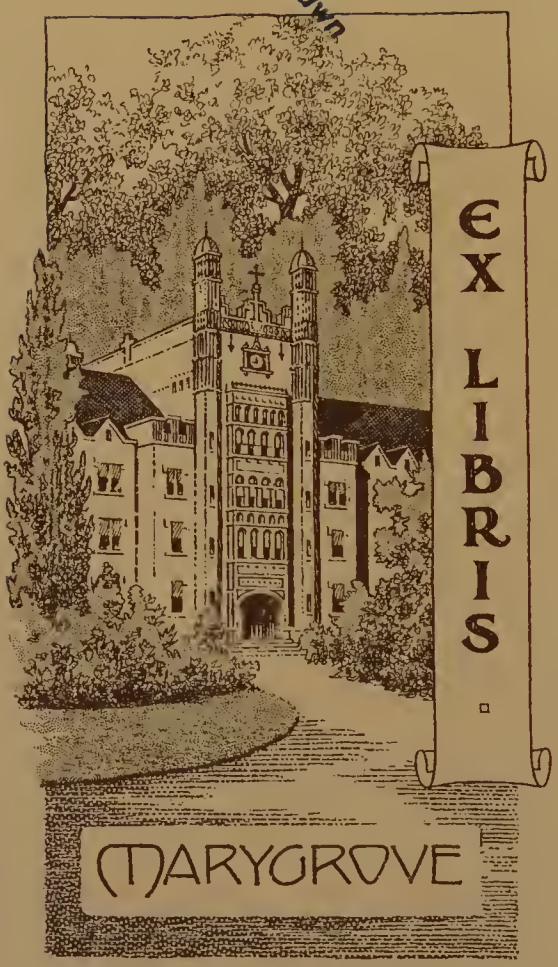

Withdrawn

THOMAS SIMPSON, MATHEMATICIAN AND EDUCATOR

His Life, Writings, and Correspondence

S I R,

I Humbly intreat the Favour of your VOTE and INTEREST to ſucceed Mr. *James Dodſon*, deceas'd, as Maſter of the ROYAL MATHEMATICAL SCHOOL in *Chriſt's-Hoſpital*; and ſhould I be favour'd with your Approbation, ſhall ever teſtify my Gratitude by a faithful and diligent Diſcharge of the Truſt repoſed in

S I R;

Your moſt obliged,
And moſt obedient Servant,

John Davis.

WE do perſonally know Mr. *John Davis* to be a Man of Sobriety and Integrity; that he has been regularly educated in the Study of the Mathematics, and has acquired an extenſive Knowledge in the moſt conſiderable Parts thereof; that he was a warranted School-Maſter in the Royal Navy during Part of the laſt War, and is now eſtabliſhed in the Buſineſs of teaching Navigation, and other collateral Branches: We therefore beg Leave to recommend him as a Perſon in all Reſpects qualified for the Place he ſollicits.

December 5, 1757.

T. Simpſon, *F. R. S.*
Member of the Royal Academy of Sciences at *Stockholm*, and Mathematical-Maſter in the Royal Academy at *Woolwich*.

Wm. Mountaine,
Mathematical Examiner to the Right Worſhipful Corporation of the *Trinity-Houſe*, and *F. R. S.*

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THOMAS SIMPSON AND HIS TIMES

BY

FRANCES MARGUERITE CLARKE, PH.D.

NEW YORK
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To
DAVID EUGENE SMITH

FOREWORD

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FRANCES MARGUERITE CLARKE.

Columbia University,
August, 1928.

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CHAPTER I

Introduction

The history of mathematics during the five hundred years preceding the eighteenth century shows that while England produced relatively few giants in the field of creative mathematics she nearly always possessed a class of able textbook makers who served to keep alive an interest in the subject.

Although Italy had the greatest and most productive mathematician of the thirteenth century, Leonardo Fibonacci, England had in Sacrobosco an author of the most popular work on the sphere that had appeared up to that time, and also of a work on Hindu-Arabic arithmetic entitled *Tractatus de Arte Numerandi*, both of which were widely used until the close of the sixteenth century.

No Italian of the fourteenth century stands out as showing real genius in mathematics. Yet England during this century produced several writers of more than ordinary ability, as for instance Richard of Wallingford, who wrote on trigonometry and arithmetic,[1] and Thomas Bradwardine, the author of four popular treatises on mathematics.[2] It was, however, in a treatise of the greatest French writer of this period, Nicole Oresme, that the first-known use of fractional exponents is found,[3] and in another treatise by the same author there is set forth a suggestion of coördinate geometry, by the locating of points by means of two coördinates.[4]

While Pacioli in Italy, von Peurbach and Regiomontanus in

[1] *Quadripartitum de sinibus demonstratis; De sinibus et arcubus in circulo inveniendo; De chorda et arcu;* and *De chorda et versa.*

De rebus arithmeticis and *De computo.*

[2] He wrote *Arithmetica Speculatiua*, Printed at Paris, 1495; *Tractatus de proportionibus; Geometria speculativa*, and *De quadratura circuli.* See Smith, *History of Mathematics*, I, 236.

[3] *Algorismus proportionum.* See Cantor, *Vorlesungen über Geschichte der Mathematik*, II (2), 121.

[4] *Tractatus de uniformitate et difformitate intensionum.*

Germany, and Chuquet in France were the outstanding mathematicians in the fifteenth century, England produced relatively few writers on mathematical subjects, and none of special prominence.

The sixteenth century was a rich one in Italy. It gave to the world the all-round genius Leonardo da Vinci; Cardan, author of a work popularly known as the *Ars Magna*, a treatise which was devoted solely to algebra and which set forth the theory of algebraic equations so far as it was then known, including the solution of the cubic; Tartaglia, who, in addition to being the first to complete the solution of the cubic, and to apply mathematics to artillery science,[5] was the author of the best treatise on arithmetic[6] of the time; Ferrari, discoverer of a solution to the biquadratic equation; and finally Bombelli, author of *L'Algebra parte maggiore dell' arimetica divisa in tre libri* in which the reality of the three roots of a cubic equation is set forth in the case in which the cube root of an imaginary expression is involved in the result secured by the Tartaglia-Cardan rule.

The sixteenth century in France was less spectacular, although Paris and Lyons were centers of mathematical activity. Viète, the greatest of all French mathematicians of the time, wrote chiefly on algebra,[7] but was also interested in geometry[8] and the calendar as well as mathematics in general. He was among the first to employ letters to represent numbers in algebra. He applied algebra to geometry in such a way as to lay a foundation for analytical trigonometry; gave one of the earliest methods of evaluating π by infinite products; indicated powers more simply than his predecessors had done, and in general added greatly to the fund of mathematical knowledge.

England was later than Italy or France in her appreciation of mathematics, or at least in her publication of works on the subject. This century did produce however the so-called founder of the English school of mathematics, Robert Recorde, whose

[5] *Nuova scienza . . .* , Venice, 1537.

[6] *General Trattato di nvmeri et misvre*, 2 parts (volumes), Venice, 1556–1560.

[7] *Isagoge in artem analyticam.*

[8] *Effectionum geometricarum canonica recensio*, and *Supplementum Geometriae*, Paris, 1593.

mathematical works, written in the English language, showed originality in treatment and in method of presentation. His treatises[9] were popular in England as textbooks for over a hundred years.

The accomplishments of Germany during the sixteenth century were as great as in Italy and France. A notable group of arithmeticians appeared in Germany at this time, Köbel and Adam Riese being two of the most outstanding. Germany also produced three strong algebraists,—Rudolff, Stifel, and Scheubel; and made an advance in geometry, in the study of higher plane curves. Clavius did a great deal to extend the knowledge of mathematics in Germany during this century, although he did little to extend its boundaries and did most of his writing in Rome.

The seventeenth century marked the development of a new spirit of intellectual internationalism. In Italy, Cavalieri contributed his principle of indivisibles which formed the basis of a crude kind of calculus; Galileo constructed a satisfactory telescope, invented the sector compass, and developed an interest in astronomy; and Viviani contributed to applied mathematics and more especially to geometry.[10] The center of mathematical activity however had shifted from Italy northward. France now vied with England as the center of mathematical activity, but as Professor David Eugene Smith remarks:

"Perhaps England's mathematics was more usable in the natural sciences, while that of France was more of the nature of *l'art pour l'art;* but any such distinction is easily attacked."[11] Such names as Harriot, Napier, Oughtred, Wallis, Barrow, and Newton balance well with Fermat, Desargues, Descartes, Pascal, Mersenne, and l'Hospital.

In *La Géométrie,* an appendix to his great treatise on method in science, Descartes presented the first published work on analytic geometry. Fermat wrote on the same subject but without publishing his material, and both he and Mersenne contributed to the theory of numbers. Desargues was the most original contributor

[9] *The Grovnd of Artes; The Castle of Knowledge; The pathewaie to knowledge; The whetstone of witte.*

[10] *De Maximis, et Minimis Geometrica Divinatio in Qvintvm Conicorvm Apollonii Pergæi adhvc desideratvm,* Florence, 1659.

[11] See Smith, *History of Mathematics,* I, 370.

of his time in the line of pure geometry. Pascal perfected the theory of the cycloid and solved the problem of its general quadrature. L'Hospital was author of several textbooks designed to introduce the new analysis into France. Roberval was well known for his discoveries in the field of higher plane curves and for his method of drawing a tangent to a curve.

England, on the other hand, had in Harriot one of the founders of algebra as we know it today; in Napier, the author[12] of two treatises on logarithms; in Briggs, a man who realized the importance of Napier's discovery and who, by exploiting the base 10, aided in its success; in Oughtred, the author of a brief treatise on arithmetic and algebra entitled *Clavis mathematicae* and the inventor of the slide rule; and in Isaac Barrow, an editor of Euclid's *Elements* and the *Data*, the works of Archimedes, the conics of Apollonius, and the spherics of Theodosius. The culmination of this productive period in English history was reached in Newton and his contemporaries,—Halley, Wallis and Gregory.

John Wallis (1616–1703) was one of the most active and original investigators known to history. The power displayed by Wallis in his famous work entitled, *Arithmetica Infinitorum*,—a preparation for the new calculi which were shortly to come into existence, can be traced to his study of the writings of classical antiquity. In this work is to be found for the first time the familiar symbol, ∞, to denote "infinity"; and here, too, is to be located the expression for π as a product of an infinite number of rational fractions. Halley contributed to the field of astronomy, and Gregory, like most of the leading British mathematicians of the period, was equally interested in physics and mathematics.

It was the influence of Newton, without doubt the greatest physicist, astronomer, and mathematician that England had known, that was now felt so as to overshadow the genius of all his contemporaries. His name is not only attached to the discovery of "universal gravitation," but also to the calculus of fluxions. A new field of geometrical research was opened up by his classification of plane cubics, and by showing the value of

[12] *Mirifici Logarithmorum Canonis descriptio . . . Authore ac Inventore Ioanne Nepero, Barone Merchistonii & Scoto,* Edinburgh, 1614; *Mirifici ipsius canonis constructio,* 1619.

Rabdologiae sev nvmerationis per virgulas libri duo, Edinburgh, 1617.

the idea of "central projection" he laid the foundation of the projective geometry of today. His *Optics* is still authoritative, while the *Arithmetica Universalis* contains many important features concerning algebra and the theory of equations. The *Principia* is, perhaps, his greatest work. It was admired by a few of his countrymen but was imperfectly understood by them, and it was left to foreign scholars to give it the study which it deserved, sometimes followed by unfair and unfriendly criticism.

While the historic period during which Wallis was the outstanding figure was one of peace between England and the other countries of Europe, that in which Newton gained a reputation witnessed one of the longest and most bitter scientific controversies in the history of human thought. English mathematicians sought to defend Newton's rights of priority to the discovery of the infinitesimal calculus, while the whole of Germany rose to sustain the claim of Leibniz to the same discovery. English readers became so influenced by the arguments as set forth in the *Commercium Epistolicum* (1712) and Raphson's *History of Fluxions* (1715), that they gave Leibniz little credit for his work. On the other hand, Leibniz was so stung by the accusations of his English critics that he too showed a spirit that cannot always be commended. The facts are that Leibniz knew of Barrow's work on the "differential triangle" before he began his own investigations, or could have known of it, and that he was also in a position to know something of Newton's work. The evidence is also clear that Newton's discovery was made before Leibniz entered the field; that Leibniz saw some of Newton's papers on the subject as early as 1677; that he proceeded on different lines from Newton and invented an original symbolism; and that he published his results before Newton's appeared in print.[13]

Newton recognized three types of the calculus. In his *Principia* (1687) he made some use of infinitely small quantities. His second method was that of fluxions, and in his *Method of Fluxions*[14] he states that "the moments of flowing quantities are as the velocities of their flowing or increasing." Newton's third method, that of limits, appears in his *Tractatus de Quadratura Curvarum*

[13] Smith, *History of Mathematics*, II, 699; Cajori, *Amer. Math. Month.*, XXIV, 145; XXVI, 15.

[14] Colson translation, p. 24 (London, 1736).

(1704).[15] The notation employed by Newton in his method of fluxions, consisting of a cumbersome system of dots, does not lend itself as readily to calculation as that employed by Leibniz. In addition Newton had favored in his treatises the geometric method of proof which set an example to his contemporaries and followers that was not always favorable. His great contribution to the theory consists in part in his extension of the method to include other functions then in common use, in his recognition of the fact that the inverse problem of differentiation could be used in solving the problem of quadrature, and in his wide range of applications of the subject.

Thus by the close of the seventeenth century, the subject matter of elementary mathematics was practically what it is today. It was during the latter part of the seventeenth and the early part of the eighteenth century, that we reach a time in England when textbooks in this field became so standardized and numerous as to necessitate the mention of only the most important. Their mission henceforth was to improve the methods of presenting theories which already were largely developed, and to adapt the application of these theories to the needs of the world. From this time on they ceased to be a great factor in the presentation of mathematical discoveries.

In spite of the impetus given by his work, the immediate followers and successors of Newton were unprepared to reach his heights. However, James Stirling (1696–1770) and Abraham de Moivre (1667–1754), distinguished themselves by extending and applying the doctrine of series. De Moivre in particular contributed much to the improvement and progress of the theory of probability, as well as to the theory of complex numbers. Newton himself remarked of his contemporary, Roger Cotes (1682–1716), that, "If Cotes had lived we had known something." He was one of the first to compile a list of formulae of integrals, and his posthumous *Harmonia Mensurarum* (1722) shows the extent of his success in this direction. No less distinguished is Brook Taylor (1685–1731) to whom we are indebted for notable progress in the differential calculus, in the integral calculus (the integration

[15] Smith, *History of Mathematics*, II, 694; Ball, *History of Mathematics*, 6th ed., 344 (London, 1915); A. von Braunmühl, *Bibl. Math.*, V (3), 355. The reader is referred to these authors for further details.

of several classes of differential equations), in the theory of finite differences, and finally for progress in the field of perspective.

Relatively little has been written of the history of mathematics in England during the period immediately following the death of Newton. The reason for this is that the materials of research offer nothing spectacular and often lie scattered among countless volumes of the transactions of learned societies and of the periodical literature of the day. A knowledge of this period of the eighteenth century is essential, however, to a complete understanding of the later development of mathematical subjects both in Great Britain and in the United States.

After 1730 the interest in mathematics, awakened by the work of Newton, was kept alive and mathematics became a popular subject largely through the medium of textbooks. England was again represented in the field of mathematics by her textbook writers who were either self-taught or had received only a limited university training. Mention should also be made of the popularity of periodical literature on mathematics at this time, which not only indicated wide-spread interest in the subject, but became in itself a popularizer of the science and thus served to keep alive this interest.

The Newton-Leibniz controversy had served to render England even more insular, in creating a patriotic adherence to Newton's ideas and methods. While the continent still sought to extend the boundaries of the science, England was content to extend the knowledge of the Newtonian philosophy and analysis within her borders. The resultant wider diffusion of the knowledge of mathematics among the English is especially well described by Ramachundra[16] in reference to this period:

"That sound judgment which gives men well to know what is best for them, as well as that faculty of invention which leads to the development of resources and to the increase of wealth and comfort, are both materially advanced, perhaps cannot rapidly be advanced without, a great taste for pure speculation among the general mass of the people, down to the lowest of those who can read and write. England is a marked example . . . there is no

[16] *A Treatise on Problems of Maxima and Minima Solved by Algebra*, by Y. Ramachundra, Calcutta, 1850. See also DeMorgan, *A Budget of Paradoxes*, 2d ed., 2 v., edited by D. E. Smith, The Open Court Publishing Co., Chicago, 1915, I, p. 375.

country in Europe in which there has been so wide a diffusion of speculation, theory, or what other unpractical word the reader pleases. . . . In all parts of England in which manufacturing pursuits have given the artisan some command of time, the cultivation of mathematics and other speculative studies has been, as is well known, a very frequent occupation. In no other country has the weaver at his loom bent over the *Principia* of Newton; in no other country has the man of weekly wages maintained his own scientific periodical. With us, since the beginning of the last century [eighteenth century], scores upon scores—perhaps hundreds, for I am far from knowing all—of annuals have run, some their ten years, some their half century, some their century and a half, containing questions to be answered, from which many of our examiners in the universities have culled materials for the academical contests. All these questions have always been answered, and in cases without number by the lower order of purchasers, the mechanics, the weavers, and the printers' workmen. . . . This country has differed from all others in the wide diffusion of the disposition to speculate, which disposition has found its place among the ordinary habits of life, moderate in its action, healthy in its amount."

It is thus due to the work of the textbook writer and the editor of the popular mathematical periodical that much of the diffusion of mathematical knowledge took place. A study of the life, writings, and correspondence of Thomas Simpson, one of the best of the English textbook writers of the time, not only reveals the status of mathematics in Great Britain during the period in question,—that is approximately from the death of Newton up to the introduction of the analysis into England in the time of Woodhouse,—but it also illustrates the fact that English writers developed a school of mathematics on lines radically different from those followed by continental scholars. It is important to observe that, during this period, English textbooks were almost the only ones in use in American schools, and American teachers were trained to a large extent by English leaders, so that the development in both parts of the English-speaking world was substantially the same for nearly a hundred years. Thus a knowledge of the eighteenth century is of particular value in interpreting conditions of the present day both in England and in the United States.

Fortunately, in this connection a great amount of manuscript material consisting of the correspondence, and mathematical and private notes of Thomas Simpson, hitherto unpublished, is to be found in the private collection of Professor David Eugene Smith, and has made possible this study. This material has been particularly valuable in illustrating the type of mathematical thought in existence in England during the century following the death of Newton, and in illustrating a general "diffusion of speculation" with regard to mathematical subjects. It has been supplemented by other material consisting of textbooks, treatises, correspondence, and periodical literature of the eighteenth century, to be found in the Columbia University library, New York Public library, the Congressional library, and the Bodleian library, as well as by material in the British Museum, and in the private libraries of Professor David Eugene Smith and George A. Plimpton. This material is so extensive both in regard to Thomas Simpson himself and in regard to the contributions of his contemporaries that it permits the appraisal of Simpson's work and also the summarizing of the chief tendencies of the times in which he lived, a study which should be of value both to students of the history of mathematics and the history of education.

CHAPTER II

Early Life

Born on August 20, 1710, and into a scientific world of which Newton was the central figure, Thomas Simpson was early intended for the trade of stuff weaving, the vocation of his father. His early life at Market Bosworth in the county of Leicester is described by Hutton,[1] a friend of his later years. At this time England was dominantly rural, and practically all industrial processes were carried on in rural villages; "the weaver, or spinner, or maker of nails, was at one and the same time the artisan and farmer."[2] Market Bosworth was a weaving settlement, and Simpson's father, though of ample circumstances, neglected his son's education, thinking to bring him up to his own humble trade. Throughout the eighteenth century the education of the child of the lower classes was the private concern of the parent. "The children of the working class were expected to gain the training that was needed for their social and economic rôle under the system of apprenticeship. The boy who was to become a weaver entered the home and employ of a master weaver and learned from him his art."[3]

The education offered to Thomas Simpson was no exception to the general rule. Thomas was only taught to read English, but possessing a rare intellect he showed an interest in study in general. He educated himself through reading the literature with which he came in contact. He taught himself to write, and made opportunities to associate with people from whom he might learn. His father objected to what he considered an undue liking for useless books, and struggled with his son to abandon such pursuits and devote his energy to perfecting his trade. Disagreement between father and son became so intense that Thomas

[1] *Memoirs of the Life and Writings of the Author*, by Charles Hutton, printed as the preface to *Select Exercises for Young Proficients* by Thomas Simpson, 1792 ed.

[2] *Nationalism and Education Since 1789*, Macmillan by E. H. Reisner, New York, 1923, p. 221.

[3] Ibid., p. 230.

felt obliged to leave his father's house and settle at Nuneaton, a small distance from Bosworth. Here he lived at the house of Mrs. Swinfield, a tailor's widow, who had a son and a daughter. The son was only two years' Simpson's senior and became one of his best friends.

At Nuneaton Simpson continued to work at his trade of weaving, at the same time improving his knowledge through reading such books as he could procure. The occurrence of a solar eclipse on May 11, 1724, was a determining factor in Simpson's life, since it served to direct his attention to scientific interests. "The phenomenon so awful to many who are ignorant of the cause of it," writes Hutton, "struck, the mind of young Simpson with a strong curiosity to discover the reason of it, and be able to predict the like surprising events. It was however five or six years before he could obtain his desire."

After Simpson had been at Mrs. Swinfield's for some time, a pedlar whose custom it was to stay at her lodging house came to Nuneaton. Although he was merely an itinerant merchant, he added to his income by means of furtune telling and by resorting to astrology. As was often the case with people who practised this calling in the small country village, he came to be regarded as almost superhuman. It was therefore natural that Thomas should seek his acquaintanceship, the result being that he succeeded in gaining both his friendship and respect. When the pedlar left to attend the annual Bristol fair he presented Simpson with a copy of Cocker's *Arithmetick* containing a short appendix on algebra and one on Genitures, by the almanac maker Partridge. With these books Simpson began his study of mathematics, gaining such a mastery of the essentials as to amaze the pedlar on his return. The story is that the pedlar then told his future fortune, and predicted that Thomas "within two years would turn out a greater man than himself."

With this encouragement Simpson began to educate himself in the art of fortune telling, and from time to time was assisted by his friend when the latter's journeys brought him to Nuneaton. He finally became proficient enough to adopt this as his profession, and aided by his teaching in an evening school, derived a sufficient income to be able to give up his weaving industry. He soon became known throughout Nuneaton, Bosworth, and the

surrounding countryside for his art in foretelling events. The nature of the work given in his evening school is unknown, but judging from his ability in mathematics and his later career in that field it is not improbable that even at this early date he conducted classes in that subject. His fame at this period in his career however rested wholly upon his ability in fortune telling. Simpson was consulted on every occasion quite like a lawyer or a physician, to advise as to marriages, business deals, future careers, and various other matters of daily life. He always declined to give information concerning such items as stolen goods, or life and death, but his success in answering what he termed "lawful questions" became so apparent that, had he not been firmly convinced of the lack of truth in the proceedings, he might have continued to follow this profession.

Owing however to an incident in connection with his career as astrologer, Simpson found it necessary to leave Nuneaton. A young lady, who wished to hear of a friend who had gone to sea, prevailed upon Simpson much against his will to give her some account of the man. Simpson tried to dissuade her through suggesting the possibilities that he might raise a ghost or a devil, but her desire and persistence were so great, that he finally agreed to oblige her. Hutton records the circumstances as follows:

"The scene of action pitched upon was the barn and the young Swinfield was to act the devil or ghost, who being concealed under some straw in a corner of the barn, was, at a signal given, to rise slowly out from among the straw, with his face marked so that the girl might not know him. Everything being in order, the girl came at the appointed time; when Simpson, after cautioning her not to be afraid, began muttering some mystical words, and chalking round about them, till, on the signal given, up rises the taylor slow and solemn, to the great terror of the poor girl, who before she had seen his shoulders, fell into violent fits, crying out it was the very image of her lover; and the effect upon her was so dreadful, that it was thought either death or madness must be a consequence."

As a result of this scene, Thomas found it necessary to abandon his profession as soothsayer. He had meanwhile married Mrs. Swinfield, who was then almost old enough to be his grandmother, being around fifty years of age, and according to Hutton "having

been looked upon as an old maid before her first marriage." Both of her children by her first marriage were older than Simpson. Their life together in Nuneaton seems to have been pleasant enough, young Swinfield following the profession of tailor, and Simpson having a sufficient income through his teaching, weaving, and fortune telling to support the family.

The tranquillity of his home life being disturbed by the unfavorable outcome of the conjuring incident and the serious condition of the girl's health after her experience, it seemed imperative for Simpson to leave Nuneaton with his family. He therefore settled in Derby for two or three years, instructing pupils in an evening school and carrying on his trade of weaving during the day. While here, at the time of the parliamentary election in 1733, he wrote a song in favor of the Cavendish family.

During this period he continued his work of self-education, and acquired such a knowledge of arithmetic, algebra, and geometry as to permit him to understand certain questions proposed in the *Ladies' Diary*. Through this magazine he learned of a still higher branch of mathematics which was much talked of at the time. This was the method of fluxions. Not understanding Latin he was unable to find a text on the subject which he could profitably read, although Hayes already had published (in 1704) a folio in English. This, however, was scarce and was, like others of its kind, too expensive for Simpson to buy. Finally an acquaintance loaned to him a copy of Stone's *Fluxions*,[4] a part of which is a translation of the Marquis de l'Hospital's *Analyse des infiniment petits*. Through a study of this book and by his own powers he was enabled to publish an even better treatise on the same subject a few years later. By the age of twenty-six Simpson had educated himself through the doctrine of fluxions, and although his education had a mathematical bias, it was thorough so far as he had gone.

Although his first publications of which we have any record were verses on subjects non-mathematical, in 1735 and 1736 he published in the *Ladies' Diary* two mathematical questions of a

[4] *The Method of Fluxions, both Direct and Inverse. The former being a Translation from the Celebrated Marquis De l'Hospital's Analyse des Infinements Petits: And the Latter Supply'd by the Translator, E. Stone,* F.R.S., London, MDCCXXX.

rather difficult nature which showed the advance he had already made in the field of mathematics. In one of these he mentions that his residence is in latitude 52°, showing that it was proposed before his removal to London.

Having abandoned fortune telling, he found it difficult to support his family, and as his wife was soon to give birth to a child, he determined to avail himself of the better opportunities offered in London, leaving his family at Derby until he could establish himself.[5]

Newton had now been dead nearly nine years but his name and influence were still in the ascendant. The mathematical world was a brilliant one, made up as it was of such men as Brook Taylor, De Moivre, Ditton, Whiston, Jurin, Robins, Maclaurin, Robert Smith, and Saunderson. By 1730 the differential notation had become well known on the continent, and the separation of the Newtonian school from the general stream of European thought begins to be observable at this time. Newton's notation was generally adopted in England and hence the common language and facility of intercommunication of ideas which are of the utmost importance to science, were denied to her scholars. Although the results obtained by Newton, Taylor, and Maclaurin were immediately translated into the differential notation on the continent, the work of the continental writers was not given like treatment in England. Such was the state of affairs open to Simpson in the year 1735.

[5] He must have moved to London before January 16, 1736, for a letter addressed to him at London bears that date. See letter below, p. 27.

CHAPTER III

Early Career in London

SITUATION AND OCCUPATION

Upon his arrival in London Simpson settled in Spitalfields, a Huguenot weaving community, and while establishing himself as a weaver, taught mathematics during his spare time. In this settlement he not only found opportunity for self-advancement but also a great demand for his teaching. Although engaged in trades and ordinary occupations the Huguenots had carried with them into Great Britain their love of intellectual activities; in fact DeMorgan refers to the settlement in Spitalfields as "one of the most remarkable proofs of the diffusion of speculation," especially among the "ordinary habits of life."[1] This is seen in their musical societies, their debating clubs, and, what is still more remarkable, in their scientific associations. It was an age of clubs throughout England, but those of the Huguenots rose above the level of mere drinking houses, and were established to foster intellectual interests. Those found among the Huguenot silk weavers of Spitalfields were especially well known.

In his private notes Simpson frequently mentions the "Mathematical Society in Spitle Market." This society flourished between 1717 and 1845,[2] and was known to possess a very good library.[3] DeMorgan's account of the society furnishes some indication of Simpson's life at that time.

He writes, "Its habitat was Spitalfields, and I think most of its existence was passed in Crispin Street. It was originally a plain society, belonging to the studious artisan. The members met for

[1] DeMorgan, *A Budget of Paradoxes*, I, 376, 377, 2d ed., edited by David Eugene Smith, Chicago, The Open Court Publishing Company, 1915.

[2] See Smiles, *Huguenots in England and Ireland*, 336 et seq.; Sydney, *England and the English in the Eighteenth Century*, I, 61.

[3] See *Encyclopaedia Britannica*, XII ed., under "Societies, Mathematical."

discussion once a week; and I believe I am correct in saying that each man had his pipe, his pot, and his problem."[4]

The rules of the society are illuminating, one of the oldest reading, "If any member shall so far forget himself and the respect due the Society as in the warmth of debate to threaten or offer personal violence to any other member, he shall be liable to immediate expulsion, or to pay such fine as the majority of the members shall decide." Their great rule however, and one which no doubt proved most beneficial to Thomas Simpson, appears printed in large letters on the back of the title page of their last book of regulations. "By the constitution of the Society, it is the duty of every member, if he be asked any mathematical or philosophical question by another member, to instruct him in the plainest and easiest manner he is able." This regulation was observed by a majority of the members, many of Simpson's correspondents adding the word "Philomath" to their signatures to indicate membership in such an organization. From an account given by one of the members, it seems that the rule applied to each member who was capable of supplying information, and he was obliged to give it, or pay a fine of one penny.[5] Such a regulation would tend in Simpson's case to enhance his reputation since his abilities as a teacher and mathematician were only in need of demonstration. Besides, with the exception of Dolland,[6] Saunderson,[7] Crossley,[8] and DeMoivre[9] no men of such

[4] DeMorgan, *Op. cit.*, I, 378.

[5] The account given by Benjamin Gomperz (1779–1865), tells of how he became a member:

"I happened to pass a bookseller's small shop of second-hand books, kept by a poor taylor, but a good mathematician, John Griffiths. I was very pleased to meet a mathematician, and I asked him if he would give me some lessons; and his reply was that I was more capable to teach him, but he belonged to a society of mathematicians, and he would introduce me. I accepted the offer, and I was elected, and had many scholars then to teach as one of the rules was, if a member asked for information, and applied to any one who could give it, he was obliged to give it, or fine one penny."

See DeMorgan, *Op. cit.*, I, 378.

[6] John Dolland (1706–1761), the silk weaver who became the greatest maker of optical instruments in his time.

[7] Nicholas Saunderson (1682–1739), the blind mathematician. He lost

outstanding abilities as Simpson possessed were associated with the society during his lifetime, although the membership was large.[10]

In such conducive surroundings, Simpson gradually gained in knowledge and became so successful in his teaching as to find it possible to return for his family and bring to London his wife, two step-children, and in addition a daughter Elizabeth who had been born during his absence. During the following year his family was further increased by the birth of a son, Thomas.

INTEREST IN THE CALCULUS

Simpson's chief interest in mathematics during the early years in London still was centered in Newton's method of fluxions. Bishop Berkeley's[11] publication, *The Analyst*, printed in 1734 attacking Newton's treatment of the subject, had resulted in several controversies which aroused general interest, and became the starting point for all philosophical discussion of the new mathematics in England during the eighteenth century. Berkeley's controversy with Jurin[12] and Walton,[13] and later Jurin's

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his eyesight through smallpox when only a year old. At the age of twenty-five he began lecturing at Cambridge on the principles of the Newtonian philosophy. His *Algebra*, in two large volumes, was long the standard treatise on the subject.

[8] Crossley was not in a class with the others mentioned.

[9] See below pp. 36, 37 and pp. 95, 96.

[10] The society gradually dwindled, and in 1845 was reduced to nineteen members, whereby an agreement was made by which sixteen members became Fellows of the Astronomical Society without contribution. Thus the old mathematical society merged into the Astronomical Society. It was then found that the old artisan element had been extinct for many years and there was not a man among the members in 1845 who was not educated enough to become a Fellow of the Astronomical Society in the usual way.

[11] Dr. George Berkeley (1684–1753).

[12] James Jurin, at one time a student in Trinity College, Cambridge, who had imbibed Newtonian teachings from Newton himself. Jurin wrote under the pseudonym of "Philalethes Cantabrigiensis."

[13] John Walton, Professor of Mathematics in Dublin.

controversy with Robins[14] and Pemberton,[15] had the effect of causing a refinement of many existing conceptions. The debate between Jurin and Robins, especially, is the most thorough discussion of the theory of limits carried on in England during the eighteenth century. As a result attention was largely directed to the subject of fluxions, and in 1736 four British text-books on the subject appeared, more than were published in all the thirty years preceding.[16]

One of these publications, written by John Muller, was brought to Simpson's attention. His early correspondence with Muller shows his difficulties in understanding the latter's work entitled, *A Mathematical Treatise: Containing a System of Conic-Sections; with the Doctrine of Fluxions and Fluents, Applied to various Subjects*, London, 1736.[17]

Muller was at this time in the ordnance office in London, but upon the founding of Woolwich Military Academy in 1741, he became head-master and later master of artillery and fortification there. In later years, as we shall see, his life was curiously connected with Simpson's. This treatise represented his first attempt in print. As there were few works of the kind in the English language at the time of its publication, his method of treating the subject was better received than the work itself warranted. He made no use of "infinitely small quantities, nor of nascent or evanescent velocities," but wrote rather of the "ratios of velocities." Simpson had of course been grounded in the subject from a study of Stone's treatise, and therefore found it difficult to reconcile the two works. Consequently he corresponded with Muller concerning his difficulties. Muller in turn tried to meet

[14] Benjamin Robins, a self educated mathematician and a native of Bath. He was a man of considerable reputation. See G. A. Gibson, "The Analyst Controversy," in *Proceedings of the Edinburgh Math. Soc.*, 1899, vol. xvii, pp. 22–25.

[15] Henry Pemberton, a friend of Sir Isaac Newton.

[16] Cajori, *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse*, p. 149.

[17] The work is dated from The Tower of London, and is dedicated to the master-general of the ordnance, the Duke of Argyll and Greenwich, although Muller's name does not appear in the ordnance lists in "Angliae Notitiae" at this period.

certain of his criticisms as the following letters from him, found among Simpson's papers, will show:

"Sir,

"I received yours in which you are pleased to send me some difficulties which you met with in reading my book: which I am very willing to explain[e] as much as it is in my Power. Your mistake[n] lyes in Supposing the force with which the wind Strikes every particle of the Curve, to be as the correspondent semi-ordinate BM, which is intirely false for how you can make it out, that the force of the wind blowing uniformly in the direction pm, should be as Bm I cannot tell: for if we conceive that

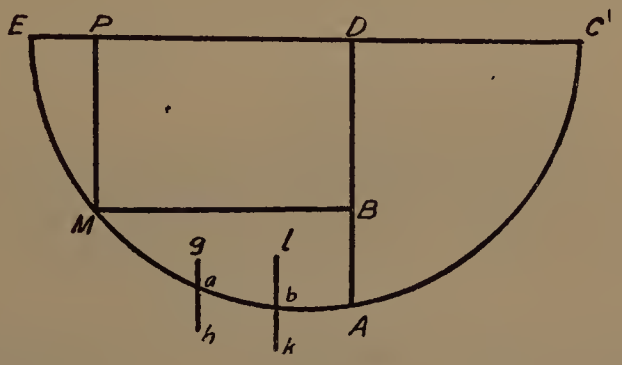

Fig. 1

all forces (acting in the direction pm parallel to DA) are constant[e], we may conceive them to be represented by an innombrable of equal lines gh, lk, parallel to DA, and whose centers of gravity a, b, are all placed in the flexible line EmA. Now if each of them equal lines are Supposed = 1, their number from A to m, will be (calling $Am(zl)$ $l \times z = z$; but we have proved (as well as your self) that this quantity must be equal to $\frac{a\dot{x}}{\dot{y}}$: therefore $z = \frac{a\dot{x}}{\dot{y}}$, or $z\dot{y} = a\dot{x}$, is the Equation of the Curve, and not as you have found $y\dot{y} = c\dot{x}$.

"I hope Sir that this and what I have said, relating to this Subject in my book, will fully convince you, of its Solution.

Therefore, having no more to say at present, as that I shall always remain

"Sir Your Most obedient Servant,
"J. Muller."

"From the Tower the 23d of July 1736."

This explanation did not fully convince Simpson for in a letter written the following August, Muller attempted an explanation of the same sort, at even greater length, and closed his discussion with the following paragraph:

"Mr. Bernoully in his *Essay de la manoeves des Vaisseaux*, proves that the power applied in any place in a direction perpendicular to the Curve, is reciprocally as the radius of Evolution in that point, which he brings out to be, reciprocally as $\frac{\dot{x}}{\ddot{y}}$, which gives the same Solution as ours.

"N.B. It must be observed that the vertical powers are reduced into perpendicular ones, by saying as the Square of the radius is to the square of the sine of the angle of incidence, so is the vertical power to the Perpendicular to the Curve, one."[18]

[18] Muller (1699–1784) was a German but *The Monthly Review*, XVIII, 31, says, "tho' Mr. Muller is no Englishman, his works are English, and he is therefore to be considered as an English author." He was born in Germany in 1699.

In addition to the *Mathematical Treatise*, which was popularly known as *Treatise of Fluxions*, Muller was author of the following works:

A Treatise Containing the Elementary Part of Fortification, Regular and Irregular . . . for the Use of the Royal Academy of Artillery at Woolwich, Illustrated with 34 copper plates, 1st ed., 1746; 2nd ed., 1756; 3rd ed., 1774.

A Treatise Containing the Practical Part of Fortification, . . . Illustrated . . . For the Use of the Royal Academy at Woolwich, London, 1755; 3rd ed., 1774.

A Treatise of Artillery, . . . to Which is Prefixed an Introduction, with a Theory of Powder Applied to Fire Arms, 2nd ed., with . . . *additions*, London, 1768; 3rd ed., 1780.

The Attack and Defence of Fortified Places, in 3 parts, 3rd ed. enlarged, also *Bellidor's New Method of Mining*, and *Vallière on Counter Mines*, 2 pt., London, 1770.

The Field Engineer, Translated from the French with Observations and Remarks, 2nd ed., corrected with additions, London, 1773.

Elements of Mathematicks to Which is Prefixed the First Principles of Algebra, 3rd ed. improved *with a New Treatise on Perspective*, 2 v., London, 1765.

A draft of what constituted Simpson's answer to Muller is to be found among the *Simpsoniana*, and reads as follows:

"Sir

"Yesterday I had ye pleasure of receiving yours; and return you many Thanks for the Willingness you have shewn to gratify my desires: But pardon me sir if I still continue to think the Mistake lyes in your own side, and that; what you have done is so far from proving ye Truth of your Assertion, that it rather evinces ye Contrary. You are pleased to say, that my mistake lyes in supposing y^e^ force with which y^e^ Wind strikes every particle of y^e^ Curve *AM*, to be as its correspondent semi-ordinate *BM;* but where is it that I have supposed any such thing? or is

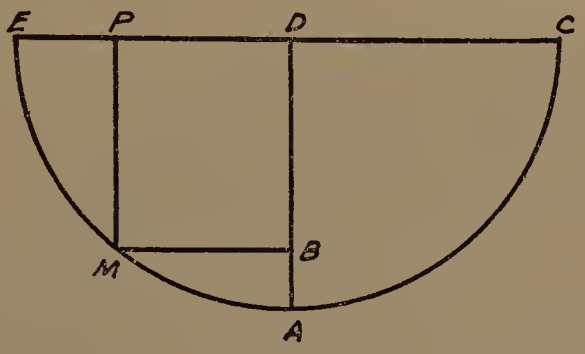

Fig. 2

there even so much as a particle of y^e^ Curve named? I did indeed assert that y^e^ sum of all y^e^ Momenta or forces acting on ye Curve *Am* would be as its semi-ordinate *Bm*, but wherein is that assertion so intirely false? If ye particles of wind move at equal distances from each other, (as is allways allowed in such cases) is it not evident, that ye sum of those striking on ye Curve *Am*, as well as ye sum of their forces, will be allways as ye correspondent semi-ordinate *Bm*? But allowing your way of reasoning, and supposing ye curve *AE* equaly divided at *m*, as many particles

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A New System of Mathematicks, to Which Is Prefaced the First Principles of Algebra, etc., London, 1769.

Muller has been referred to as "the scholastic father of all the great engineers of this country employed for forty years." A fairly complete collection of his works is to be found in the British Museum.

of wind would pass betwixt E, and p, as betwixt p, and D; but wheather thus wou'd be consistent, or not, I appeal to your own better Judgment; and at present only

"beg leave to subscribe myself,
"Sir your (realy) most
"Obedient Servant,
"Tho–, Simpson."

"P.S. before I conclude I must beg your pardon for ye liberty I have took since no other motive induced me thereto but y^t of gratifying ye desires of some Gentlemen (your subscribers) who had solved this same problem long before your Book appeared; for I am so far from pretending to Criticise on your Book, that I freely own, I never yet looked on it, more than once."

Simpson's letters seem to indicate that he was not particularly interested in Muller's *Treatise of Fluxions*, and consequently its influence on his own work would appear slight, his own testimony being that he had not looked at it "more than once." As his own treatise on fluxions was then in progress, however, one would be more inclined to believe that Muller's method of fluxions influenced him considerably, particularly in the light of other evidence showing his knowledge of Muller's work to be more thorough than he admits at this time. A book of *Essays* published in 1740 by Simpson contains "an easy way to reduce a compound fraction to a simple one." For first "hints" on the subject Simpson acknowledges indebtedness to Muller's "*Treatise on Conic Sections and Fluxions.*" It is highly probable that these thoughts occurred to him at this time of his correspondence with Muller, and if so, Muller's thoughts on "Fluxions" may also have shaped his own.

SIMPSON'S FIRST PUBLICATION

Simpson's abilities became so well known and the number of his private pupils increased so rapidly that he was encouraged to publish by subscription, *A New Treatise of Fluxions: wherein the direct and inverse Method are demonstrated after a new, clear, and concise Manner, with their Application to Physics and Astronomy: also the Doctrine of Infinite Series and Reversing Series universally, are amply explained, Fluxionary and Exponential Equations solved: together with a Variety of new and curious Problems.*

Although Simpson had experienced many interruptions in its preparation, owing to the necessity of spending much of his time in earning a living, this treatise appeared in 1737.

At the time of its publication there were few English textbooks on the subject of fluxions, and these rarely included the practical applications which Simpson thought necessary. Very little had been attempted up to that time with infinite series, so that the work was sufficiently original even though Simpson generally followed Newton's method. His textbook marks a departure from Newton in his definition of fluxion for Simpson makes it a finite distance and Newton makes it a velocity. The work gives some indication of Simpson's ability as a teacher of mathematics as well as of his own personal progress in the subject, his presentation being so clear and direct as to adapt it to students of ordinary ability. The second part, which treats of infinite series, contains excellent demonstrations to accompany every statement, and has illustrations in the form of easy examples. He also presents various new rules concerning the doctrine of series. The third part contains a familiar method of finding and comparing fluents, together with easy and practical applications. The fourth part shows the use of fluxions in physics and astronomy, and gives some interesting material relative to the "doctrine of Pendulums and Centripetal Forces." The supplement, which was added to this treatise, is a collection of miscellaneous problems which have no bearing on the other four parts, the following topics being considered: an investigation of the areas of spherical triangles; curve of pursuit; paths of shadows; motion of projectiles in a given medium; the manner of finding the attractive force of bodies of different forms, acting according to a given law; and "divers others in different Parts of the Mathematicks."

Simpson embodied in this work the substance of his researches to date. The emphasis placed on practical applications of mathematics reflects the attitude of the century. Interest in fluxions had been awakened by the greatness of Newton as well as by the national feeling aroused through the Newton-Leibniz controversy. Textbooks such as this were in demand by men throughout England who either desired to educate themselves or to seek instruction from the class of private teachers to which Simpson

belonged. This latter class exerted a great deal of influence in the first half of the century. For example, it is said of Abraham de Moivre that after his reputation had been established he spent a great part of the day at Slaughter's Coffee House in St. Martin's Lane where he was ready to answer any question on the method of series for the fee of one guinea. Similarly James Dodson, great-grandfather to Augustus De Morgan, educated himself in the subject of annuities and the doctrine of chance, and earned his living, before his appointment to the Royal Mathematical School in Christ's Hospital, by answering questions proposed to him on this subject; and John Landen, regarded by the contemporary continentalists as ranking with Newton and Taylor, was not only self-taught but is also said to have taken private pupils for a time. There were many others of less fame and less genius whose work met the same need, being not only specialists in general mathematics but also in particular branches. Thus Simpson was at this period looked upon as an expert in the method of fluxions.

All this goes to show that mathematics was becoming democratic. The universities were educating a chosen few who could afford the expense; the dissenting academies often gave a more extensive course in science than the universities, but were chiefly concerned in educating dissenters; special schools, such as the military and naval academies, were primarily for sons of gentlemen; while the great public schools paid little attention to the scientific side of education. It was in the democratic movement sponsored by the self-taught and private teachers that mathematics was coming to the front.

CORRESPONDENCE AND CONTRIBUTIONS TO PERIODICAL LITERATURE

Simpson as an exceptional teacher was well known in London, but Simpson as a textbook writer on such an important and advanced subject became known throughout all England. Many wrote to him asking help on certain questions which appeared in the *Ladies' Diary* or in *Merlin's Magazine*, or on some difficulty encountered in reading a current text. Few names of prominence appear among Simpson's correspondents of this early period but the letters illustrate admirably three characteristic practices of the eighteenth century, namely, the "correspondence system of

instruction" in mathematics, the interest and general reliance of the public on periodical literature for material, and finally the practice of carrying on disputes of more or less scientific interest in such periodicals, and the attention which such quarrels aroused.

Between the years 1714 and 1744 Henry Beighton was nominally editor of the *Ladies' Diary*, but as a matter of fact Anthony Thacker took care of the mathematical part of the magazine while Mrs. Beighton managed the rest. As a contributor to the periodical Simpson became acquainted with Mr. Beighton, but seems to have been on even more friendly terms with Thacker. This appears from the fact that the latter, in 1739, speaking of a problem which Robert Heath had accused Simpson of pirating, mentions the fact that he and Simpson had worked the problem out "six years previously during an all night session."

Thacker seems to have made special use of his correspondence with Simpson, however, to discuss matters relating to the *Ladies' Diary*, and to secure Simpson's help in getting it ready for the press. The following extract from his later correspondence[19] shows the high esteem in which he held Simpson's opinions:

"I have got a transcript of your solution to the seventh question but have not had time to compare it with mine, and as their is no other of Mr. Beighton's correspondents who have done anything like it in Solution; it makes me more pressing for your Opinion &c. I would have sent you Merones Solution to the 4th Question but have no time now to transcribe it, . . . I have been to Oxford with Mr. Beighton to see Dr. Bradley[20] & one of Ld. Digby's Relations &c and I have got a very large acquaintance . . . please send me a line how I may know the best method of finding the longitude[21] at Sea and the title of that Book you think best on Navigation. . . . "[22]

Beighton also seems to have consulted freely with Simpson in

[19] Letter dated June 4, 1742.

[20] A celebrated astronomer of this century.

[21] Simpson replies to this request that there is no definite method except by computing it from the course and distance. Harrison's work was then in progress in response to a prize offered by the British government.

[22] Simpson recommended "Willson's," probably *Elements of Navigation*, by William Wilson.

matters concerning the *Ladies' Diary* and he recognizes the latter's help as follows:[23]

"Mr. Thacker (who the Inclosed old Letter will Mention) Continues here in the place I live at, Is Settled in a School; whose Stay I was not a Little Solicitous for; In whose Company I have spent the greater part of the Leisure time for a Year; He has made very great advances in the Higher parts of the Mathematics &c. *And much of it from ye help of Those Lights you have favoured the World with and Set forth to shine amongst us of an Inferior Orb. . . .* "

The greater part of his letter is given over to a discussion of the prize question in the *Ladies' Diary* for the year 1741, which Thacker had answered; to a comparison of the three solutions sent in by Mr. Robertson,[24] Mr. Davy, and Mr. Ward,[25] respectively, to the 219th question, "To investigate a theorem for finding the solid content of any hoof of the frustrum of a cone;" and to a statement of the fact that neither he nor Thacker had arrived at a correct answer to the forty-second problem in the *Universal Arithmetick* of Sir Isaac Newton. He closes with a request for Simpson's opinion on all of these problems, indicating that Simpson's mathematical ability was fully realized by men of their standing.

During this period Simpson proposed and answered many questions in the *Ladies' Diary*, sometimes under his own name as in the years 1735 and 1736, but later by such fictitious names as Hurlothrumbo, Kubernetes, Patrick O'Cavanah, Marmaduke Hodgson, Anthony Shallow, Esq., and perhaps others.

His chief interest in the *Ladies' Diary* during these years was, however, that he found it a vehicle for his answers to charges of piracy made against him by Captain Robert Heath, from whom he had received letters as early as January 15, 1736.[26] These letters are of no especial importance except to show the highly supercilious character of the man with whom Simpson had to deal in open conflict through periodicals. Most of Heath's

[23] Letter dated Griff, July, 1741.

[24] Robertson, *A Complete Treatise of Mensuration*, 1739, p. 160.

[25] Ward, *The Young Mathematician's Guide*, London, 1706, pp. 414, 415.

[26] Addressed to Mr. Tho. Simpson, An Eminent Mathematician in London.

charges appeared in *Merlin's Magazine*, which to judge from his own condemnation, was a journal of little scientific value. Of this periodical Heath writes in one of his early letters to Simpson, "I beg you'll not do anything to expose the weak Brothers of the Under Class, who Assemble to improve one another in the Oracle,[27] if you don't lend them a ray of Your Genius to further their conceptions."[28]

He goes on to say in the same letter, "I see you have given Mr. J. Turner an Answer to his Queries in *Mr. Urban's Magazine*, and propos'd a Herculian Problem for the *Philomaths* to solve, or Mr. T., if he likes it. I have so far a taste for Mathematics, as to be in Love with all the Learned of that Noble Science! Their Performances are Treasures. . . .

"I have two or three Masters, &, if you will own me for a Scholar Likewise, I can with Reason say, I have the best Instructions. . . . "

Whether Heath can be taken at his word when he says that "there are but small numbers of Men in the World, whose chief delight is Science in comparison of the vast numbers, whose chief delight is to degrade & turn Mathematicks into jest and ridicule; and consequently to expose or demean the professors of the subject as mad men . . . ", is doubtful, and in all events, an exaggeration. It is true that at this time Berkeley had already published *The Analyst*, and such men as Swift, Arbuthnot, Steele, Addison, and other literary authors had often published composite sketches satirizing different subjects and people of the day, among them mathematicians and mathematics.[29]

[27] Merlin's Magazine.

[28] Letter to Simpson dated January 16, 1736.

[29] In later years when Simpson was editor of the *Ladies' Diary* an interesting article was written by him concerning the habit of the literary men of the time to satirize mathematicians. In this particular case he referred to Mr. Pope's lines "And running round ye circle finds it square," and discussed whether Pope appeared to have had a "right Idea, of what Geometers mean, *by squaring of ye circle* or of ye Branch of Science against which he more than once levelled his Satyr: and whether, in general, it be consistent with ye Character of a Great Genius, eminently excelling in his particular field, to subject his understanding to ye censure of persons, greatly his inferior in natural abilities, either by an unbecoming pride, to be thought possessed of all valuable Learning, or to decry and turn to

Heath closed his second letter to Simpson with the remark that he had heard that De Moivre's principles in the *Philosophical Transactions* were very good on problems dealing with the laws of chance, but that he would prefer to be shown a short cut by Simpson and also to receive a copy of his book.[30] He mentions also that Muller's book entitled a *Treatise of Fluxions*, "is so dear that many are afraid to buy it."[31] To Heath's suggestion that Simpson help him in his mathematical study, the latter replied that he had declined all "paper correspondence" because it took up too much time and also frequently resulted in ingratitude and trouble.

NATURE OF THE CONTROVERSY WITH HEATH

The trouble which Simpson anticipated came to him in another form. In this instance a Mr. Parkinson accused Simpson of plagiarism with regard to certain problems, and immediately a "paper war" was begun. Robert Heath attacked Simpson, while Dean Eagland and Mr. Holmes supported him.

The quarrel was long and abusive but such quarrels were as characteristic of the times as the publication of scientific matters in popular journals. People who knew Parkinson said that Heath had written the accusations but had hired Parkinson to sign them with his name. Simpson intimates the same in the answer which he printed in *Urban's Magazine*, and in a letter which he wrote to Merlin, editor of *Merlin's Magazine*. A letter from Anthony Thacker to Simpson, gives an excellent account of the controversy from his point of view:

"Sheffield July 19, 1739.

"Mr. Simpson Sir—

"It is no small pleasure that I have this opportunity to send you these lines by my worthy friend Mr. Roebuck hoping thereby to here of your good health and proceedings in your ingenious

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Ridicule, what, he makes appear (at his own Expence) he had not taken y[e] trouble to understand."

See De Morgan, *A Budget of Paradoxes*, 2nd ed., II, 276, 277.

[30] Since this letter is dated March 14, 1736/7 he must refer to Simpson's *Treatise of Fluxions*.

[31] This complaint was also made in *The Monthly Review*, the work selling for ten shillings while the average sold for five.

and learned studies & about, a month since I was with Mr. Turner of York of whom I'm sure wou'd have been rejoiced with the same opportunity & showed me the Gentleman's magazine and Monthly Oracle & in the latter some enemies which I knew was pretended friends when I was in London &c. But as the most part of those patched up stories by Mr. Heath & Parkinson were such as you know I was assured of to be false &c And which (I think) I've shewn to Mr. Turner to be nought but the result of malice and Ill nature &c. the greatest area that can be inclosed any number of given right lines which he makes so much noise about was not you and I acquainted with it 6 years since and did not I prove to you that the question by Mr. Lovatt &c must be inscribed in a circle viz the given lines chords of that circle whose diameter was the line sought. And did not I stay with you all night and you demonstrated the same for any no. of given lines &c. The same falsehood I know of other tales which those invented to serve there purposes &c but I have not time to trouble you with any more at present &c. I think your book will sell not one pin worse than if they had as much in praise of the author as they'd said against him &c. I've some time past disposed (or at least caused to be disposed) of a great no. of your books and continually keep recommending and disposing of above 17 of the Ashbey's and about half as many of Marls and all with in the space of one year &c. Mr. Travis' Question in the present Diary I think is composed from the consideration of such problems as at page 35 in your book &c.

"I should be glad to have your opinion of it and the 2nd &c. So wishing you all health and good encouragement from all those ingenious souls whose sublime thought and active fancy lead to the study of the mathematics &c.

"From yours in haste,
"Ant. Thacker."

"P.S. Sir at your leisure I shall be very proud to receive a letter from you &c please to direct to your humble servant A. T. to be left at his House in Shottle near Wirksworth Derbyshire."

The charge of "piracy" against Simpson, was in the course of the controversy enlarged to include all his failings and shortcomings. In an answer to such charges, Simpson wrote various elaborate papers, denying that others had written letters for him

to sign, denying that Eagland wrote his book on "fluxions," denying that he received six shillings a week for "mastering to-gether" the book, and finally very frankly admitting that his English and spelling were defective. "But," he says, "I never in my life proceeded upon false principals as I know of in ye solution of any problem, indeed must own that in the solution and composing of the first of those manuscripts of mine which they have in their possession, which was five or six years ago, I'd made but small progress and that the spelling too was defective. Nay I had so bad an opinion of them myself above three years since that I copied some of them, and gave him [Mr. Parkinson] the originals, others I gave him without thinking them worth while copying but believing they would be of service to him, and others of later date he copied himself without my knowledge. And I appeal to the Reader if it is not hard that I should be both reproached and abused for my good nature. . . . *But supposing my Knowledge in Letters to be no greater now than it was at that time what is that to our present Debate?* or why must I be censured and derided on that account? *did I ever affect to appear Learned, or to Make use of French and Greek phrases out of old Dictionaries (as some other persons have done).* No! I *am not ashamed to confess that I have benefited myself more that way in these four last years, since my being in Town, than I did all my life before, and he may advertize it again if he pleases.* I am not quite so ignorant as to be incapable about answering his little Query about the scrap of Latin so meanly and impertinently proposed; and if I do not advance farther it will be only owing to my own Incapacity tho' indeed my progress at present is so small, that there are many places in this Learned Gentleman's Letters which I cannot make English of.

"And as to dullness, I shall readily acknowledge my manner of writing to be as dully insipid, as he pleases, rather than to enter into a Dispute with him upon that Account; . . . "

CORRESPONDENCE CONCERNING THE DISPUTE

Most of the letters which Simpson received at this time are either full of this dispute or ask for help on some problem occurring in his own textbook and in the current numbers of the periodicals.

T. Glaspoole of Aylsham wrote of some malignant verse which

Heath had published; W. Lighton of Hertford asked for assistance on problems in Simpson's text; William Piper of East Worldham near Alton thanked him for advice as to the inadvisability of buying Cotes's *Tables* from which the constructions and demonstrations are omitted, asked about the dispute, and enclosed an answer to a problem occurring in the *Ladies' Diary* for 1739; one J. Claxton sent an answer to the attacks made upon Simpson in *Merlin's Oracle*, volume 2, page 460, which he desired to have published in *Urban's Magazine* or in *Merlin's;* and finally William Knight of Truro, Cornwall, signed himself "an unknown friend," and asked aid in the study of Simpson's book.

INTEREST OF MATHEMATICAL CLUBS IN THE DISPUTE

Such a quarrel would appeal to the average man more than a mathematical discovery for it was something spectacular and was easy to discuss. The mathematical clubs found such material of great interest, as is shown in a letter from Christopher Mason of Eastbourne, Sussex:

"Sir

"I have been for some time very ambitious to trouble you with a letter but expected my business would have permitted me to come to London this Xmas which it will not; & seeing your gallant defence in *Merlin's Magazine* I can not omit any longer but congratulate you upon your good success in defending both truth & your own Character against the vile aspersions, misrepresentations, and low scurril language of a haughty insolent and envious person, who it is very apparent will stop at nothing to do you a prejudice & gratify his own malicious temper.

"I was lately at a Mathematical Club in Lewes, where every member greatly rejoiced at your elegant defence & drunk towards your good success not only in this but in all your undertakings; . . . One of their Members appears in *Merlin's Magazine* by ye name of Philotherus, & designs to appear in ye next *Ladies' Diary* (if permitted) under ye same Name; And as there are several Questions in ye ensuing years Diary, and sometimes in ye Magazine, that may be solv'd by methods, Rules, & Problems, in your Ingenious & useful *Treatise of Fluxions*, I made a proposal to ye before mentioned Club, that either they or myself solving

such problems to quote proper places in yr. Book, which proposal was agreed to wth yr approbation, wch I think is but doing you Justice, & expressing ye false Aspersions of R. H. . . .

"The 6th Qu: in the *Diary* is very applicable to problem: 6th, page 168, in your incomparable *Treatise* which you may further observe where mine is only particular, & yours universal. 'Soon after a hard gale of wind &c,' See *Ladies' Diary* pa: the 20th, Qu: the 6th. . . .

"The 3rd Qu: in the *Diary* my operation being something long and intricate I have not here room to insert it, but shall make bold to trouble you with it in my next; neither can I so well apply it to any one problem as to form, but it is rather a mixture of several. I shall be very proud to receive a line from you with your opinion of this spiral question & heartily wish you the common compliment for the present season & remain, Dear Sir,

"Your most obedient and humble servant,

"Christopher Mason."

"Eastbourne Sussex"
"December 22, 1737."

Simpson himself was a member of one of these mathematical clubs,—of the order of *Philomaths*. While such clubs were common in England at this time, the members were not primarily mathematicians but earned their livelihood in some business during the day and regarded this study as an avocation. The highest ambition of a great majority of the members was to appear in print as having proposed or solved a problem. The *Philomaths* was perhaps the best known of this type of clubs, the members making it an aim of their organization to answer enigmas appearing in journals.

John Turner of York, in a friendly letter[32] to Simpson discussing various contributors to magazines, wrote of one of these clubs:

"As to Mr. R. F's business he is a writer in ye Custom house[33] at Hull being ye Comptroller Clark & his first appearance in ye Mathematical way about 8 years ago in y^e^ *Ladies' Diary*. He has

[32] Letter dated September 26, 1737, York.

[33] Simpson wished to know of his occupation because he was a mathematical contributor to the *Ladies' Diary*.

two brothers, One at Hull & another at Wibach, a good ingenious young fellow who sent up a solution to Mr. Perryam's Questior Signed I. F. mentioned in ye last Magazine for Aug. page 504. We have few or none in this party who have made any considerable advances in ye Mathematicks, & therefore I do not think it worth while to give you their names.

"There is one Robert Heath who lives at Foulsham in Norfolk who was lately in yr. Company at London at a *Mathematics Club*, & passed under ye Name of Sr. Michael Man who is terribly enraged at you. He pretends you have Pyrated from him ye Method of inscribing Polygons in a circle, in order to make ye Areas a Maximum. . . . "

Simpson exchanged a number of letters with Turner on various mathematical subjects, and regarded him as not only one of his finest pupils, but also as one of his best friends. They discussed together an explanation of the rectification and quadrature of an exponential curve; a justification of Simpson's *Treatise of Fluxions* against certain of Turner's objections; and explanations or solutions to problems occurring in the periodicals.[34]

Among letters written to Simpson is one from a *Philomath* named Thomas Anderson of Preston, Lancashire, near the New Chapel in Fryergold. The letter shows how many of the members of these clubs felt the need of help, and how much such meetings and correspondence must have meant to them. In fact the origin of the societies is to be found in group meetings in local communities for discussion of mutual difficulties in the study of mathematics, long before there arose a popular desire to answer problems in journals. This desire for instruction perhaps best appears in the letter itself, for Anderson wrote:[35]

"Mr. Simpson Sir:

"This comes to you hoping your Goodness will Excuse me for troubleing you with this letter and hoping you will Grant me this Small Request which is easily done by you whom is so well

[34] See letter dated March 7, 1742, in which Turner styles himself, "Your pupil."

[35] Letter dated July 9, 1741, addressed to "Mr. Thomas Simpson, Mathematician. To be left at his lodgings in Crown Court near Uper Morefields, London."

acquainted with The Doctrine of Fluxions If it please you sir to Reason to me why this $\sqrt{xy} + yy$ or of $\overline{xy + yy}^{\frac{1}{2}}$ its X^{ns} is by your Theorem $\frac{1}{2} \times \overline{xy + yy}^{-\frac{1}{2}} \times y\dot{x} + \dot{y}x + 2y\dot{y}$ now this next I cant understand why that said fluxion should be Equal to this viz $\frac{y\dot{x} + x\dot{y} + 2y\dot{y}}{2\sqrt{xy + yy}}$; if once I could understand this & another thing which I will mention by and by I could in a short time become a master of that art. I have all the Authers Concerning this: viz. *Smith pamphlet*[36] as you say in your Treatise, and *Ditton*,[37] *Stone*[38] and yours, which they all write to the Learned; if once they whould write to unlearned & I as well as the unlearned then I might have understood it by this.

"So Sir hopping you will express to me how these or such like things are expressed, viz. in page 6, 8 proposition to the Art. 85, which I cant express; and I dont know how to Read. Which I hope you know what I mean; if I could Read any part of it I could express it all, viz. here is a part of it you lack in the above proposition,—

"That is, if

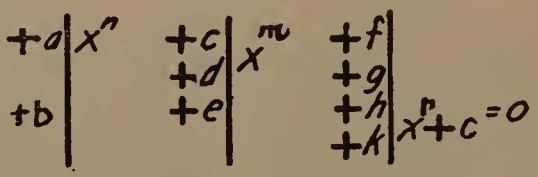


Fig. 3

If you would tell to me what means these / strokes, and the Reason why you put them in the above order I shall be bound in duty to pray for such a whorthy Gentleman as you are.

"So this is all from your Humble Servant,

"Thos Anderson, Philomath."

[36] See Cajori, *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse*, p. 165, for an account of *A New Treatise of Fluxions*, by James Smith, London, 1737.

[37] *An Institution of Fluxions*, London, 1706. For an account see Cajori, *Op. cit.*, p. 44.

[38] *The Method of Fluxions, both Direct and Inverse; The former being a Translation from the Celebrated Marquis De l'Hospital's Analyse des Infinements Petits: And the Latter Supply'd by the Translator*, E. Stone, F.R.S., London, MDCCXXX. See Cajori, *Op. cit.*, p. 50.

"Note: You may direct for me thus, % Thos Anderson near the new Chappel in fryergold preston, Lancashire. So sir I Humbly beg you would answer this in a post or two."

Such letters show, as no histories ever do, the state of mathematics in England at that time,—the ignorant seeking for light and the leaders giving attention to the trivial.

By the end of the year 1742 discussion of the Heath-Simpson controversy dropped out of the letters and a new quarrel was soon to take its place. The controversy had been a particularly trivial one, and, as far as Heath was concerned, was decidedly abusive. Heath's attitude was well summed up in a statement he once made to Mr. Beighton, that "if such eminent persons as Sir Isaac Newton and Leibniz should differ about discoveries it is small wonder that people of lower rank should cavill."

A LETTER FROM AMERICA

Among Simpson's correspondents was Alex. Buller, a Quaker of Philadelphia. The United States inherited its first mathematics almost wholly from Great Britain, and it was not until the early nineteenth century that a change of influence was felt with the influx of French mathematics. During the eighteenth century the colonies were especially dependent on England for their textbooks and in some measure for the training of their teachers. Even the textbooks published by American authors were imitations of the English until after the War of 1812. Buller, who was at one time a student in London, was the author of an early American algebra "and other parts of the Mathematicks,"[39] and the following letter, which he wrote to Simpson October 27, 1741, indicates to a certain extent the reliance which he felt on England in general and on Simpson in particular:

"Philadelphia, Octor 27th 1741.

"Friend Simpson:

"I fear y^{t} my long silence may have occasion'd me by this time to have slipt thy Memory abt. 3 years and half ago I got an insight into some difficult parts of Mathematicks from thee and

[39] See Lao G. Simons, *Introduction of Algebra into the American Schools of the Eighteenth Century*, Department of Interior, Bureau of Education, Washington, 1924, Bulletin no. 18, p. 71.

take Liberty to inform thee y^t I am in very good business here but not in ye Mathematick way. If The Gentleman y^t brings this should want any instruction or request thy assistance in buying any Books please to give him the information for ye Sake of thy old Friend, Alex. Buller."[40]

A month after writing this letter to Simpson, Alexander Buller advertised in the following manner in the *Pennsylvania Gazette:*

"November 5, 1741. Writing, Arithmetic, Merchant's Accounts, Navigation, Algebra, and other parts of the mathematics are taught by Alexander Buller, at the Public School in Strawberry Alley" . . . (Repeated November 12, 19, 26.)

Buller had received permission in October, 1738, to teach mathematics, among other subjects, in the "Publick School" (William Penn Charter School).[41] Yet according to his own statement to Simpson he was not thus employed in October, 1741. His algebra was published in 1763 by W. Jones, New York City, and was advertised in the *New York Mercury* April 25, May 2, November 21, 28, 1763.

Publication of "the nature and laws of chance"

A subject made popular by Abraham de Moivre also claimed the attention of Thomas Simpson, and in 1740 the latter brought out a treatise entitled *The Nature and Laws of Chance*, in 4 volumes. To this work he annexed *Full and clear Investigations of two important Problems added in the 2d. Edition of Mr. De Moivre's Book on Chances, as also two New Methods for the Summation of Series.*

The usefulness of different kinds of series in facilitating computations in many questions of a practical nature is purely an eighteenth century development. Yet while there was a general interest in the subject of *Chances* few treatises were published. Writing in 1753 James Dodson comments, "When Huygens, Monmort, and Bernoulli, De Moivre, and Simpson have been

[40] Addressed to "Thomas Sympson, Mathematician in London. To be heard of at the Angel Inn in Bunhill Fields. In care of Mr. Jos. Wragg."

[41] Minutes of the Publick School, 1712–1770, Vol. I, p. 26.

See Simons, *Introduction of Algebra into the American Schools of the Eighteenth Century*, Department of the Interior, Bureau of Education, Washington, 1924, Bulletin no. 18, p. 71.

named the other writers on the subject must be searched for among the transactions of the several royal societies and other miscellaneous and periodical works."[42] De Moivre published the first treatise in English on the doctrine of chances[43] in which we first get the theory of recurring series, the theory of partial fractions which Cotes's death had left incomplete, and the rule for finding the probability of a compound event. As it was then customary to publish results and conceal methods we are left without knowledge of many of the steps which led De Moivre to his results. It was, however, Reverend T. Bayes who first made use of the inverse method. His work was printed in the *Philosophical Transactions*, volume liii, page 370.

It was the desire of Thomas Simpson, as he himself expressed it, "to bring down some of the best and most useful things already known to the level of ordinary capacities. . . . " He has added several of his own ideas, however, and hastens to assure the reader that he does not wish him "to conclude from hence, that he has nothing more to expect in these few sheets, than a bare collection of low matters, or that, like some of our present writers . . . should be poorly ambitious of appearing the author of a performance, that would, was every bird to claim his own feather, be stript as naked as the Jay in the fable."

The contents of this work are comprehensive. Simpson has included, "the solutions of several abstruse and important problems; the Doctrine of Combinations and Permutations clearly deduced; a new and comprehensive Problem of great Use in discovering the Advantage or loss in Lotteries, Raffles, &c.; a curious and extensive Problem on the Duration of Play; Problems for determining the Probability of winning at Bowls, Quoits, Cards, &c.; a Problem for finding the Trials wherein it may be undertaken that a proposed Event shall happen or fail a given Number of Times; a Problem to find the Chance for a given Number of Points on a given Number of Dice; Full and Clear Investigations of Two Problems of Mr. De Moivre's, one of them allowed by that author to be the most useful on the Sub-

[42] Preface to *The Mathematical Repository*, London, 1753, v. 2, p. vii.

[43] See *The Doctrine of Chances;* or, *a Method of Calculating the Probabilities of Events at Play*, London, 1718, with later editions in 1738 and 1756.

ject but their Demonstrations omitted by him; Two new Methods for the summing of Series."[44]

In spite of the fact that De Moivre was the only other author of an English textbook on the subject, public opinion was not exactly in favor of a new publication. De Moivre held this field of mathematics almost wholly to himself. Simpson sensed this possibility and spoke of it in his preface:

"I have perhaps as high an opinion of that learned author's productions, as any of those gentlemen," wrote Simpson, "But does it follow, because one person has done well on this subject, that nothing further can be necessary?"

The work is on the whole well written. The first proposition is of use throughout, and Simpson advises an especially thorough knowledge of it. The problem of finding the trials necessary for foretelling that a proposed event shall happen a given number of times, is one which was of great interest to his contemporaries. The problem of finding the chance of a given number of points on a given number of dice, and its sequel concerning the duration of play, were not only regarded as difficult problems, but were also problems to which Simpson had given a great amount of research. The two problems taken from the end of De Moivre's latest edition which Simpson included, had been presented without solution by De Moivre, and therefore Simpson's treatment of them was a real contribution. Both of these last mentioned problems, numbered the twenty-fourth and the thirtieth in Simpson's text, were regarded by him as especially important.

CONTROVERSY WITH ENTERPIPHULUS

It was shortly after this publication that Simpson found himself a participant in another controversy in which the name of De Moivre figured. Several letters, which are evidently drafts intended for an editor of one of the periodicals, are found in the unpublished Simpson material. These have to do with his quarrel with Enterpiphulus, a contributor to one of the magazines, who had questioned Simpson's method with regard to series. In defense against certain accusations Simpson stated that his methods were no more "liable to objections" than were

[44] See the new edition, London, 1792.

those of De Moivre, which was misinterpreted by his antagonist to mean that Simpson "willingly insinuated that the Learned Mr. Demoivers Series was liable to the same objections as were made to his." Simpson defended himself against the latter accusation in the following manner:

"I never said or in the least insinuated that either of them were liable to any objections at all but only took the liberty to shew him that I might with as much justice carp at that Great Man's Performances, because his series which Enterpiphulus has unjustly brought into debate, would be of no use in solving this equation $bx^2 + cx^3 + dx^4 + c = h$ as he has done at mine for y^e same or like reason although they have no relation at all to it; and both fully answer the fields for which they were calculated. . . . I shall assert that, not only Mr. Demoiver's Theorem but every other performance whatsoever is liable to the very same objections which he has brought to mine . . . and I must tell him he has been guilty of a breach in decorum in bringing that learned gentleman's performances into dispute, and taking upon himself to vindicate that worthy gentleman's work instead of his own wild assertions. . . .

"In another place he is pleased to express himself thus, "whereas by T.S.'s rule (or series) is but a perticuler Case of that in ye 9th section of Dr. Cheyn's *Fluxionum Methodus Inversa;*" here is inconsistency indeed! He in his other letter positively asserts ye series to be expressed in General Terms; and makes that his objection; And here it best pleases him to have it but a perticuler case of Another series; which series itself is very far from being general. . . . "

This anonymous disputant would seem to have had some connection with Robert Heath, judging from the fact that Heath threatened several times to silence Simpson "by sending his letters up to Enterpiphulus."

CORRESPONDENCE RELATIVE TO THE LAWS OF CHANCE

After the publication of Simpson's treatise, *The Nature and Laws of Chance,* he was soon in communication with many people concerning questions in this subject. Early in 1742, John Muller wrote:

"Sir: A friend of mine has send me a question in chance which

he is very desirous to have answered but as I have not considered that subject much I can not resolve it; I should therefore be very glad if you would do me the favour to resolve it for me; the question is as follows.

"A Person proposes to grant a farm of an estate on 2 lives viz. on the life of A (42 years) and of B (9 years) and to the longest liver of them the tenant to pay a small yearly reserved rent, and on executing the grant to Pay a fine according to the value of the improved rent.

"Query I, How many years is the chance of the life of the longest liver of the above persons to be reasonably computed at.

"2d., How many years Improved Rent maybe a reasonable fine for such a Grant?"

"If your time can permit it I should be very glad to see you here at woolwich.

"I am sir your most humble servant,
"John Muller."

John Watts of Comb Abbey near Coventry, who describes himself as a man who studied mathematics "to satisfie his curiosity," and "as much as in him lay to promote others," wrote asking the answer to the following question:

"If two Geometers of equal skill agree to play to-gether 'till 10 games are won and lost for 5 Guineas a game, What may a Spectator venture to give upon an equality of Chance for what either of them may remain winners of after the play is over?"

His reasons for writing are based upon the high regard he feels for "proficients in Mathematics" to which class he assigns Simpson, and the high regard he has for Simpson's texts on the method of fluxions and on the laws of chance. Watts was a self-educated man who, as he himself said, "made no advantage of mathematics, being employed in quite a different way of life." Although he was unknown to Simpson at this time except through correspondence, he later became acquainted on more friendly terms.

Throughout the century there was great interest in the subject of chance and its practical applications. It was through the widespread interest in the nature and laws of chance that the whole subject of insurance developed so rapidly at this time.

CHAPTER IV

Friendship with Francis Blake

publication of essays

During the same year in which Simpson published his treatise entitled *The Nature and Laws of Chance,* he brought out a volume of *Essays on several curious and useful Subjects in Speculative and Mixed Mathematicks;* in which the most difficult Problems of the first and second Books of Newton's *Principia* are explained; in 4to. In order to fully appreciate the origin of these essays it is necessary to read the correspondence between Francis Blake and Thomas Simpson, which fortunately has been almost wholly preserved. It was to Blake that Simpson dedicated this volume, as follows:

"Sir,

"As our private Correspondence occasioned my Drawing up several of the following Papers, I thence claim a sort of Right to address them to You: But I well know the common style of Dedication would to You be highly offensive; therefore all the Use I dare make of this Opportunity, is, to declare Myself to be Sir, Your most Obliged, Humble Servant, Tho. Simpson."

In the first essay Simpson discusses the theory of the apparent place of the stars, commonly called aberration, arising from the progressive motion of light, and of the earth in its orbit; and he also includes practical rules for computing the same. The practical rules were communicated by Dr. Bevis, a friend of Simpson's and also a very well known astronomer. This paper was completed and together with Dr. Bevis's rules a few copies were printed before Christmas in 1739. Simpson was very anxious to bring this matter to the attention of the public because early in 1740 a paper by Monsieur Clairaut on the same subject found its way into Great Britain among a volume of Memoirs of the Royal Academy of Sciences for the year 1737, printed at Paris. The

matter of dates was of importance in determining the independence of his own thinking on the subject.

In the second essay Simpson considers the "motion of bodies affected by Projectile and Centripetal forces," together with a discussion and investigation of most of the important matters included in the first book of Newton's *Principia.*

The third essay is a solution of the so-called Kepler's problem, or how, as Simpson remarks, "from the Mean Anomaly of a Planet given, to find a true place in its Orbit, by three several Methods."

In the fourth essay Simpson considers the motion and paths of projectiles in the resisting mediums, and he includes the most important parts of the second book of the *Principia* in relation to this subject.

In the fifth paper he has discussed the "resistances, velocities, and times of vibration, of pendulous bodies in Mediums;" whereas the sixth is an essay containing a new method for solving all kinds of "algebraical equations in numbers," which in Simpson's opinion is a more general method than anyone heretofore has given although it is based upon the method of fluxions, and some objections might be raised to it for that reason.

The seventh, eighth, and ninth papers relate chiefly to the original work of other of his contemporaries but as all three deal with subjects of great popular interest at the time, and are necessary to a thorough understanding in other parts of the book, Simpson felt they should be included. He sought merely to give a clearer explanation of the subject matter than other authors had done.

In the seventh paper he discusses the *method of increments,* of which examples are included. The eighth paper contains a concise explanation of a theorem for finding the sum of a series of quantities by means of their differences; while the ninth paper contains an explanation of a general method for investigating the sum of recurring series.

In the tenth paper Simpson has given a new and general method for finding the sum of any series of powers, whose roots are in arithmetical progression, a method which he recommends as "sufficiently general to apply to series of other types." In the eleventh essay, however, he discusses another topic entirely,

where he speaks of angular sections and gives what he terms "some remarkable properties of the Circle." The twelfth essay, which has been mentioned in another connection, is an easy method of reducing a compound fraction to a simple one, which first suggested itself to Simpson when he read Muller's "*Treatise on Conic Sections and Fluxions.*"

The thirteenth and final essay in this volume, contains a "general quadrature of hyperbolic curves." This problem had interested the leading mathematicians of the day. None of the solutions published prior to Simpson's were more than particular cases, except one which had appeared in the *Philosophical Transactions* without demonstration (number 417), and was the work of Mr. Klingenstierna, professor of mathematics at Upsal. Simpson investigated the subject by two different methods, and has rendered the "general construction abundantly more simple and fit for practice" than other writers had done at this time. It would seem that Klingenstierna was well pleased with what Simpson had done for in later years when he was appointed secretary of the Academy at Stockholm, he was instrumental in having a diploma given to Simpson making him a member of that learned society.

The essays summarize the extent of Simpson's investigations by the year 1740, as well as indicate the topics of popular interest during this period. The fact that the first application of the Newton-Raphson process in the solution of transcendental equations is made by Simpson in this volume is of historical value.[1] The way in which these essays were suggested to Simpson is exceedingly well shown through his correspondence with Blake.

CORRESPONDENCE WITH FRANCIS BLAKE

The correspondence between Francis Blake and Thomas Simpson is particularly interesting. The eighteenth century was one of unsurpassed brilliance in the development of the art of letter writing. Not only were the letters of such men as Addison, Steele, and Swift distinguished for their style but they often proved the origin of many of the masterpieces produced by these men. It seemed characteristic of the period for an author to invite

[1] *Cajori, A History of Mathematics*, 2d ed., New York, 1919, p. 227.

through correspondence criticism of a work under construction. Nor was this true only in the field of literature. Science progressed in much the same way. It was not uncommon to find two men exchanging ideas on mathematical subjects which were in the course of events incorporated into a treatise of some sort. Of such a nature was the correspondence between Blake and Simpson.

Blake,[2] a resident of Herrington near Durham, was both a patron and student of Simpson's. Little is known of his life aside from the facts that he was born in 1708, was made F.R.S. in 1746, and died in 1780. Blake rendered active support to the government during the rebellion of 1745, and was created a baronet May 3, 1774. During the period of correspondence with Thomas Simpson he seems to have engaged in business. Four papers by him appeared at one time or another in the *Transactions of the Royal Society;* one which was read May 23, 1751, was a discussion of *Steam Cylinders, Best Proportions for Them,* while another, read on May 7, 1752, was entitled *Spherical Trigonometry Reduced to Plane.*

His correspondence with Thomas Simpson between 1738 and 1740, reveals that his influence on the mathematical thought of his time was even greater than has been supposed. The earliest letter from Blake to Simpson to be found among the *Simpsoniana* is dated February 2, 1738/9[3] and is a discussion of a water wheel which Blake was anxious to perfect. He writes,

"Dear Sir—

"I perfectly understand your ingenious manner of estimating the force of water wheels, and should trouble you no more with that subject did not their different constitutions, viz, some with buckets allowing water to descend from one to another (as you have 'em), some with close buckets, and others again with only pallets) render particular calculations necessary in every case.

[2] Raine, North Durham, pp. 314, 316; Betham, *The Baronetage of England, or The History of the English Baronets and Such Baronets of Scotland, as Are of English Families; with Genealogical Tables; And Engravings of Their Coats of Arms. . . .* Printed by Burrell and Bransby, for Longman and Rees, and Robinsons, 1801–05, iii, 439.

[3] The letter was directed to Mr. Thos. Simpson, Teacher of Mathematicks in Crowncourt at the end of Long-Alley next Hog-Lane.

In executing my own project, I am confined to an under shot wheel, and a very small one being sufficient to answer the end proposed viz to supply a bathing Room with Water, I think to use pallets is the most advantageous way possible; but whether the following be so or not I leave you to determine. . . . "

Blake proceeds to outline by means of an illustration and a lengthy explanation the process he has in mind.

"These," he says, "are contrivances which appeared beneficial to me; but if there be any direction to impell the fans than the horizontal direction or if they stand not close enough together, or can be thought to receive an additional power by forming an

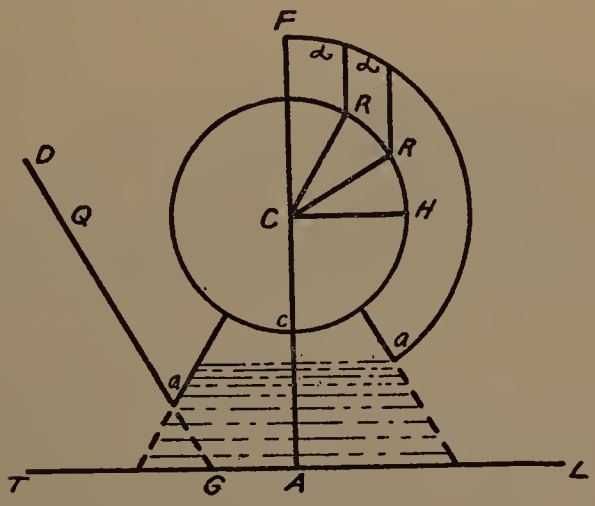


Fig. 4

angle with the radius, or lastly if any advantage be gained by buckets in this situation where the fall exceeds not two feet, I hope you'll take the trouble of informing me. Upon the supposition however that my observations were right I was at pains to compute the power of a wheel in these circumstances, thus . . .

"This doubtless is a very ill contrived wheel, but as it is commonly in use especially in these parts, I could wish you wd. employ a vacant hour in calculating its' power and the aforesaid position of the lines *CR*, *Rr*.

"When I know you are at leisure, I shall beg your assistance in some other philosophical and mathematical matters, but not until then; for, however, agreeable this correspondence may be

to myself, I cannot in any perfection enjoy that pleasure which may give you pain. For this reason pray use me with such unbounded freedom as real friendship will all ways allow; command me when I can in any respect serve you ever remembering that I admire your genius and would use my utmost power to cultivate it.

"The success depends entirely upon your own application; and I shall fairly own my disappointment if you produce not something extraordinary. . . .

"I remain,

"Dear Sir,

"Your oblig'd Friend and most

"Humble Servant,

"Francis Blake."

"Herrington Feby 2. 1738/9"

As an after thought, Blake added the following problem which appealed to his curiosity:

"P. S. Since this letter was written I rec.^{d} the following Problem; and as it has made some noise hereabouts, I beg your opinion of it.

"Prob:

"To find the weight of an infinite Pillar of Marble the diameter of whose base is 6. . . . "

In answer Simpson professed delight in such assurances of friendship, and remarked that Blake's work "neither wants for perspecuity nor ellagance." Although Simpson appreciated the construction of the under shot wheel, he found it necessary to point out to Blake an error "committed where you multiply the height of the pillar of water by the superficial content of all the 12 palletts by which the required force is 12 times as great as it really ought to be if I understand you aright. . . . "

The problem proposed in one of the magazines or, as Blake expressed it, one that "has made some noise hereabouts," was the source of discussion in Simpson's next letter to him. A topic common to all correspondents, however, was a consideration of Sir Isaac Newton's theories, and in a letter written March 10, 1738 Blake shows his interests to be in accordance with the times. "The following problem," writes Blake, "is one of the particu-

lars I proposed in my last letter to lay before you. It is an attempt to ascertain a *planetary orbit* more easily than Sir Isaac has done; and the method occurred to me upon perusing the Great Man's chapter concerning the Invention of Orbits.[4] I submit the process to you without daring to say one word in its behalf." The solution he presented was such as to show the confusion of one not thoroughly master of his subject, but at that same time was indicative of originality in thought. From this discussion Blake continued to a reconsideration of the water wheel, and, in the same connection, a study of the resistance of fluids and the momentum of bodies in mediums. In answer Simpson wrote the following interesting letter:

"Honoured Sir,

"Had ye course of our correspondence been so barren and unpleasant as not to afford the least speculation or hint for new Improvement; and had I engaged therein with no other Motives than what are meerly mercenary, your last generaous favour had over ballanced ye utmost pains and labour I have taken to oblige you: As such repeated Instances of Benevolence and Liberality could not fail to affect a Mind less susceptible of Gratitude—suffer my sincere Thanks for this, as well as ye preceding Tokens of your Favour and Generosity.

"Your articles on ye resistance of fluids afford me great Satisfaction, being not only agreeable to my Meaning but exceeding clear and expressive; tho' I cannot deny but what you since favoured me with on ye same subject, is somewhat conciser and less liable to Objections. What your Sentiments are, concerning ye Weight and Momenta of Bodies, do not indeed appear so clear and some difficulties you seem to labour under which I shall endeavour to obviate; but first it may not prove unnecessary to define, in a plain manner what is meant by those Terms.

Weight, then (as I take it) is that Force or power by which a Body, at rest, endeavours to change its place and is continually urged towards ye Earth's center. But ye Momentum of a Body, is ye Force by which it endeavours to change its place when in Motion, and equal to that power which is just sufficient to destroy its Motion in that point. . . . But the gravity or Force by which an equal Body, at Rest, is urged to change its place is no greater

[4] See *Principia*, Chapter IV.

than a single Impulse, because Weight is considered without any regard to Time and is ye same at every Instant. Hence I think it prety plainly follows that ye Momentum of a Body can in no wise be measured by its Weight but it must be understood that I speak only of ye pure force and not at all in relation to the Greatness of pressure it is able to produce which, as I said before, depends intirely on ye Texture and Degree of Hardness of ye impinging Bodies. But then supposing Y^e^ Momentum of a Body whose Weight is 10 pounds and Velocity 32 Feet per pound to be 10×32, you want to know what denomination this product must have (if it may be called so many pounds etc.) . . . we can have no idea of ye measure or greatness of any thing but by comparing it with other things of ye same species; and to ask how many feet and inches equal 3 pounds weight would certainly be ye greatest absurdity. . . . Next to find the efficacy of Water &c. . . . "

Simpson concludes with a discussion of the weight of water falling on a wheel and admits his own difficulty in understanding the way "to find the angle that the pallets must make with the radius to carry down most water to the horizontal line," his difficulty being due either "for want of understanding Blake's true meaning or not being acquainted enough with the construction of ye wheel."

Correspondence was by no means without its difficulties, and the next two or three communications from Simpson to Blake were concerned with Blake's misinterpretation of his meaning. Simpson was obliged to reword a passage that had led Blake to a belief that he thought water condensable, or might "be brought to take up a smaller space than it naturally occupies."

"I indeed thought," writes Simpson, "that it was but very little owing to ye Density of a Medium that a Body more dense than itself did not enter it, but to ye close contact and cohesion of its particles, which when a Body was driven violently against them, being suddenly and very much compressed, had not time to make way for another &c. By compresure of particles I meant no other than a pressure on every side, but whether that be improper signification I leave you to determine, which in matters of this kind are greatly superior to me. You will grant that it is possible (if I may use that expression) for a body to strike on

ye water so as to be reflected into open air again and move a considerable distance afterwards. Because it has been well attested that even cannon balls have been reflected from ye surface of ye Sea to kill several (afterwards) by the impetuosity of their Motion; and the making of Ducks and Drakes by children in some measure confirms it. Nay I have heard it said that a musket-ball by being discharged upon ye surface of water has been beaten flat, all which great effects can scarcely be conceived to happen without a violent pressure of part of the fluid.

"The incommensurability of pressure and percussion you say has started new difficulties, and that you shall be utterly at loss to calculate ye percusive Force of a Body, unless it may be valued in pounds weight and so forth; and desire to be informed how ye same can be otherwise measured,—I answer by comparing those forces one with another as all things of ye same species and as Weights themselves are measured in ye manner of doing which I am sure you need no information. . . . "

Simpson evidently favored Blake with his own researches on "the orbits of the planets," after a consideration of what he termed Blake's "ingenious attempt therein." Of further endeavors on the part of Blake to arrive at "ascertaining a planetary orbit more easily than Newton had done," Simpson wrote, "As to my own part I must confess that tho I looked on ye thought as a very lucky one and spared no endeavours to prosecute ye same yet after all have ye disatisfaction to tell you, it is intirely out of my power to produce any thing therefrom but what involves ye common Methods and is much more complicated than any of them."

On the other hand Simpson's work on the same subject impressed his friend Blake more favorably than any contemporary work, as a letter written by the latter from Twisel on June 14, 1738 would indicate:

"Dear Sir,

"Your Answer to my Letter is so full and satisfactory, that, I hope, a close attention to what is there deliver'd will enable me to surmount the Difficulties complain'd of. I'm greatly pleased, likewise, with your Calculus of the Orbits of the Planets: it carrys that matter farther than my Papers thereupon go, and

will, doubtless, be serviceable to me in reading Sr Isaac's *Principia.* Your Friendship encourages me to attaque that illustrious Author as you can clearly obviate the obscurity of his Demonstrations and so open to my view the Beauty and Order of the Universe. 'Tis true, indeed an adequate return is not easily made to such Kindnesses: but permit me, Sir, to declare my sincere Regard for you. I highly esteem your Merit, and wish I could in, any becoming manner, patronize your Genius.

"The Two following problems have employed my Thoughts pretty much for some time past. The 1st. is propos'd in this Year's *Ladys Diary;* and tho' I had the assistance of *Mathematicus Caledonius,* I don't quite approve of the solution given. The 2d. Problem is the Product of my own Fancy, but I am also dubious y^t my manner of solving it is lyable to some objections. I beg your Opinion concerning them.

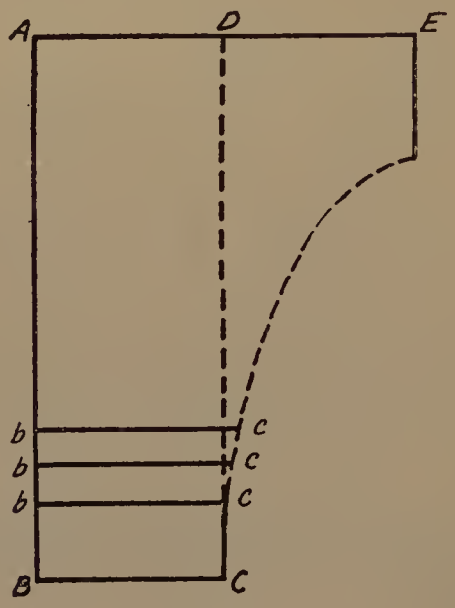


Fig. 5

"Prob. 1.

"*AB* is a Pendulum 20 feet in Length. At the Instant of its Beginning to vibrate a Power at *A* draws it up with an uniform Velocity viz at the Rate of 5 feet in a minute, y^t so in 4 minutes the Ball may reach the Point of Suspension *A*. I demand how many Vibrations will be made.

"Solution.

"*BC* is put for the vibratory Celerity of a Pendulum = *AB* = 20 f. long; and, as the angular Celeritys of Pendulums are in reciprocal subduplicate Ratios of their Lengths, the Ordinates *bc*, *bc* are taken, *sine-fine*, in those Ratios to the abscissa *Ab*, *Ab* &c to express the encreasing Celerity of the Pendulum as it shortens; and the whole Area *ABCE* expounds the sum of these constantly increasing Velocitys. Again, the Area *ABCD* expounds the Sum of the angular celeritys of a Pendulum *AB* in 4 minutes. Now, as the number of Vibrations perform'd in any assign'd Portion of Time is directly as the angular Velocitys, I presume the number of Vibrations perform'd by the variable Pendulum will be the n^{r} perform'd, in the same Time, by the invariable Pendulum, as Area *ABCD*:Area *ABCE*::1:2.

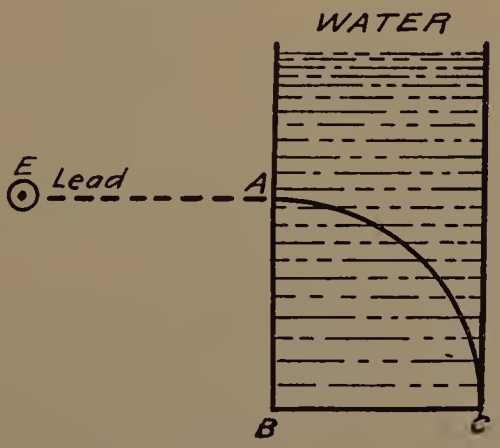


Fig. 6

"Prob. 2.

"A sphere of Lead of a given Diameter, projected horizontally, enters a Body of Water in a perpendicular Direction, I require the Velocity the sphere must move with, y^{t} the Amplitude in the Water may be a maximum, and the Equation of the Curve made by the sphere's Path in the Fluid. That is I demand the Velocity of *E* y^{t} *BC* may be a maximum, and y^{e} Equation of the Curve *AC*.

"The Impetus of the Lead against the Water is = *E* $\times^{d}$ by the simple Velocity, but the Resistance of the Fluid is = (I

presume) the Density of the Fluid $\times^d$ by the Square of the Velocity. Therefore the Problem is determinate, as the Velocity may be taken so Great that E will not enter the water at all. In my Attempts to solve the Problem, I endeavoured to find the Velocity which wd. make the Fluid's Resistance = the solids Impulses. Then I suppos'd y^t *velocity* diminished by a Degree expressible by an unknown Quantity, and putting in that Case, an Expression for the Resistance in a Form similar to the other, I subtracted the last from the first by y^e algebraical sign, and made the whole a Maximum: from whence I hoped by Fluxions to find the value of the unknown quantity, signifying the velocity sought. But I shall not trouble you with the Process at large, seeing there is enough done already to convey my meaning if the Principles are sound, and I expect it in a better Form from you.—I am,

"Dear Sir

"Your Reel Friend, and most

"Humble Servant,

"Francis Blake."

"Twisel June 14. 1738"

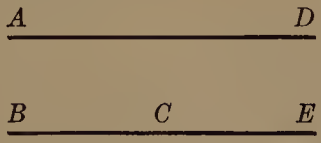
"P. S.: "If it is convenient for you to favour me soon with a Letter, you may direct it as before; otherwise, be pleas'd to direct for me at West Herrington near Durham."

This second problem of Blake's own manufacture Simpson thought "scarcely seems practicable for want of farther experiments to determine the tenacity of the fluid, and the quantity of resistance arising from the cohesion of its particles." During the next few months tenacity of fluids became a topic of interest to both men. The following quotation from a draft of a letter written by Simpson is interesting in this connection:

"Honoured Sir,

"Your Objections to what I asserted in my former concerning the motion of bodies in a fluid I have well considered: but do not find ye least cause to alter my Opinion; and therefore must believe that you understand me in a sense contrary to what I intended. You observe, my opinion is, that, was there no tenacity of parts, a body even exceedingly rarer than ye Fluid it falls on would augment its amplitude, as ye Velocity is augmented; whereas according to Sir Isaac Newton and Gravesande, you say, the

principal Resistance of a Body moving in ye same or an uniform Medium is as ye square of ye Celerity. But how you can suppose this repugnant to y^e^ foregoing I cannot conceive; because it serves as manifest Proof to confirm it. For, let 2 equal bodies enter or begin to move in y^e^ same medium from y^e^ points *A*, *B*, along the Right Lines *AD*, *BE*,



and let the velocity of that moving from y^e^ point *A* be y^e^ greater; then, because y^e^ resistance that this body meets with, or ye Decrease of celerity which is always proportional to it, by ye laws of Motion, is as ye Square of ye Velocity, it is manifest that there is a Point *C* somewhere in ye Line *BE* where ye Velocity of this body becomes equal to that with which ye other body enters ye Fluid at ye point *A*, and therefore ye amplitude of ye body entering ye Fluid at *B* must, exceed the amplitude of ye other Body by ye Distance *BC;* but this excess will indeed be something greater. Because by ye former Body's moving thro' ye medium from *B* to *C* a Motion is communicated to ye particles lying before it; which therefore lets impede ye Body in its progress when it overtakes them than if it was to impinge on them when actually at rest, as is ye case with ye other Body at its first enterance into ye Fluid. . . . That pressure and percusion should be commensurable is intirely inconsistent with Modern Philosophy; For if ye quantity of Motion or ye percusive Force be as ye Quantity of Matter drawn into ye Velocity, the Force of a Body having no Velocity at all will be nothing in comparison to that of a Body in Motion; tho' ye Force of Pressure in several cases seems to be vastly superior. . . .

"The following Theorem to find ye sum of a series of powers being one of my newest productions, and more extensive than any thing I have hitherto seen upon ye subject, I have taken the liberty to submit to your perusal.

Theorem. . . . [5]

[5] See *Essays*, Paper 8.

"If my Method of investigating this can be any ways acceptable please to let me know by your next, and thereby give me the opportunity of shewing my willingness to oblige,

"Honoured Sir,

"with ye greatest esteem,

"Your most obedient servant,

"T. Simpson."

In a subsequent communication Simpson offers Blake a solution to the problem which has, in his own words, "been the subject of a very great expence and labour and for a long time the grand topic of mathematical conversation, that is, the determination of the ratio, and true dimension of equatorial and polar diameters of the earth and a confirmation of Sir Isaac Newton's Theory from the Observations of the French Astronomers."

In connection with his study of Newton, Blake became interested in the theory of curvature, but Simpson believed certain of his ideas to be erroneous and wrote to him: "nor does what you remark from Sir Isaac Newton in the least serve to remove the obstacle; he shows indeed how to describe an ellipse thru 3 points &c but does not by this undertake to prove that it must necessarily be an ellipse and no other curve, that can pass thru those points, since the truth is there may be innumerable kinds of curves besides that which may pass thru the same 3 (or 4 or 5) points: and it not being the kind but the position and species of a given kind of curve that Sir Isaac proposes to determine. As to what you observe from Mr. Varignon's *Treatise of Curvature*,[6] it is ingenious enough but of no great importance only holding in cases where the body is urged by a force every where acting in the direction of parallel lines, so in the case where the trajectory comes out a parabola it is on supposition that the earth's surface is a plane, and that gravity acts everywhere perpendicular to it; the curve actually described by an uniform gravity directed to a center being an algebraic one for which we have not a name. The construction (or resolution) of equations relating to curvature in which you desire to have a more perfect knowledge, is not only to you but to everyone a most difficult part of mathematics; there being few equations in comparison that can be at

[6] See Smith, *History of Mathematics*, I, 470, 471.

all confirmed (but by infinite series &c.) and even in these few no general method can be observed; and therefore no general directions given in relation thereto. . . . "

Simpson added to this discussion solutions to two problems which Blake had found difficult, one being the thirty-ninth proposition in Newton's *Principia*,[7] and the other the eighth problem in the supplement to Simpson's own book on the method of fluxions.[8]

Newton's method for finding orbits continued to be a topic of interest to Blake, as did his desire "to ascertain a planetary orbit more easily." Finally in May, 1739 he seems to have arrived at a solution which met with Simpson's approval, as well as his own. His desire was to achieve an improvement on Newton's work, for in referring to Newton's method, he says, "our author's seems very intricate and tedious."[9]

Beginning with a letter written by Blake to Simpson on May 8, 1739, the two men entered upon a thorough discussion of the method of fluxions which proved to be a most productive line of thought for Blake. It was consideration of the work of Newton that really led Blake to a study of the fluxional calculus and from thence to the publication of an anonymous essay on the subject in 1741, which proved to be one of the clearest and best discussions of fluxions that appeared during the eighteenth century. Unfortunately, the essay has been little known, a fact which may be due in some part to its brevity, it being only sixteen pages in length. Blake attempted to establish Newton's *Method of Fluxions* upon more obvious principles which could be better understood by the lay mind. Simpson mentioned the essay in his *Doctrine and Application of Fluxions* (1750) and claims indebtedness: "my obligation I owe to a small Tract, intitled, *An Explanation of Fluxions in a Short Essay on the Theory;* printed for W. Innys: Wrote by a worthy Friend of mine (who was too modest to put his name to that, his first Attempt) whose Manner of determining the Fluxion of a Rectangle, and illustrating the Higher Orders of Fluxions I have, in particular, follow'd, with little or no Variation."

[7] See Newton, *Principia*, 2d ed., edited by Roger Cotes, Cambridge, 1713, p. 112.

[8] See *A New Treatise of Fluxions*, London, 1737.

[9] See Newton, *Principia*, 2nd ed., p. 64, prop. 21, Liber I.

In the light of these facts it is interesting to watch in the correspondence between Blake and Simpson, the development of the essay.

Writing to Simpson in May, 1739, Blake speaks of fluxions as follows:

"Of their truth and actual existence in nature I have entertained no sort of doubt since your instructions therein were first delivered to me; and my speculations on that subject are continued only with a view of removing as much as possible all attraction from it. I found the reasoning upon the supposed accuracy of this following.

"Definition:

"The fluxion of an quantity (viz variable) is proportional to the spaces that can be described in a given time, uniformly with the amount of velocity the flowing quantities have acquired, or remainder (at the instant of finding the fluxion) by means of any acceleration or retardation, if affected by either.[10]

"Problem:

"The fluxion of any uniformly generated quantity x, being $\dot{x}$, I require the fluxion of a quantity that is always as its square

$$\underline{A \quad x \quad B} \qquad \underline{C \quad xx \quad D}$$

that is, as xx in any cotemporary position.

"Solution:

"A, flowing uniformly towards B and C describing with an accelerated velocity, spaces towards D in y^{e} same time ever as the Squares of y^{e} other: The former may properly be represented by the Time, and the latter by the Spaces descended in that time by an uniform Gravity. Now it is demonstrated if the Acceleration of Gravity should cease, the Body would by the Velocity already acquired pass over uniformly in the same Time a space double of the other; y^{t} is, in any cotemporary Position the uniform Celeritys would become x and $2xx$. Therefore $x:\dot{x}::2xx:\frac{2xx\dot{x}}{x} = 2x\dot{x}$ the Fluxion of xx. And again, if $2x\dot{x}$ (now in y^{e} nature of a Rectangle) be suppos'd a Flowing Quantity, its' Fluxion is $2\dot{x}\dot{x}$ (if $\dot{x}$ be constant) or $2\dot{x}\dot{x}$ $2x\ddot{x}$ (if $\dot{x}$ be variable)

[10] He defined fluxion as *a finite distance*, as did Simpson.

the 2d. Fluxion of *xx;* in wch Instance Nature admits not of an higher Order, as is also evident from an uniform Gravity.

"Such (Sir) are my Ideas of this matter, which if just, may easily be extended to any Example; and tho' they prove erroneous, some Advantage may even thence accrue to myself, as you will know where the mistake lyes, and be able to point it out to, and correct it in

"Dear Sir,
"Your Real Friend and
"oblig'd Humble Servant
"Francis Blake."

"P. S. I shall inclose this to Mr. Story with an Order to him to pay you five Guineas, which, I beg you'll receive."

This letter marks the real beginning of the formation of the essay on fluxions.

Simpson meanwhile was carrying on the different researches that were later to be included in his book of essays. Blake was progressing in a study of the method of fluxions, and at the same time was giving Simpson counsel on the material that was to prove to be the substance of the second paper of his essays. The following interesting passage from a letter written by Simpson in 1739 is in answer to some objections Blake had offered concerning his method of treating the "problem of ye Apsides:"

"The difficultys occuring to you in ye perusal of my Problem of ye Apsides I humbly apprehend are not, as you imagine, from ye want of a sufficient knowledge in Astronomy to enter upon the subject but to your mistaking or confounding it with ye precession of ye Equinoxes, which has no relation thereto; . . . The cause of this, your mistake, by your Queries seems to be from reading Sir Isaac Newton's Section upon ye Motion of Bodies in moveable orbits; these moveable orbits of his not being analogous to anything in nature, but a mere artifice to find ye Apsides; but the *method he takes* upon this Occasion, not to mention its difficulty, is perhaps, y^e^ *least elegant of anything that can be found in that great Man's performance. . . .* "

Some of Simpson's ideas concerning *orbits* were communicated in his next letter along with his approval of Blake's idea and interpretation of the method of fluxions:

"Honoured Sir,

"Your last has effectually removed the Difficulties in ye former, and I now not only see clearly your Meaning but much approve of ye Invention; which when the uniform space is proved to be ye accelerated space as mx^m to x^m will certainly afford a more clear and familiar Idea of Fluxions than anything extant; but to do that with the accuracy and rigour that is requisite in a Matter of so much importance, will I fear be found more difficult than you seem to apprehend; it being yet out of my power to produce any better way of effecting it than that in my last Letter.

"As to ye Method mentioned in my last concerning ye Invention of Orbits, when I came actually to apply ye same since ye arrival of your last it did not appear to answer so well as I expected; and therefore instead thereof I have taken y^e Liberty to substitute y^e following, and y^t rather as it is most agreeable to what we have already done in a particular case of y^e subject.

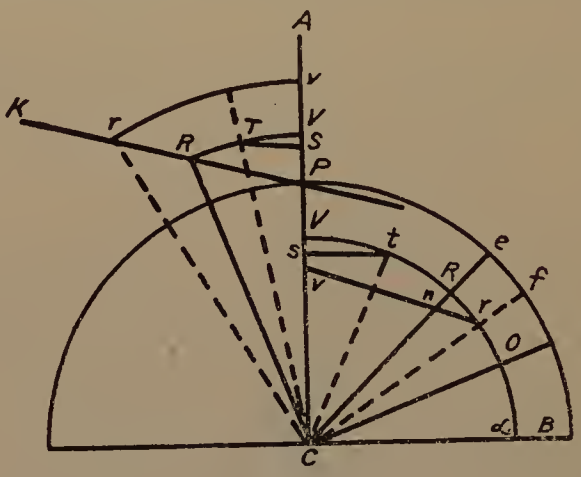


Fig. 7

"Problem:

"A Projectile is let go from P at a given Distance PC from C, y^e center of Force, in a given Direction PB, with a given celerity; To find its Trajectory &c according to any law of centripetal Force; the absolute Force at P being given. . . . "

Simpson's ill health interrupted his correspondence for a time, but he continued to suggest topics of investigation for Blake,

such as those concerning *loci, series, planetary orbits, Kepler's problem,* and different problems arising in connection with the reading of the first and second books of Newton's *Principia.*

"The Demonstration which you favoured me with to my last proposition," wrote Simpson, "I like much, it being more concise than my own, which I had Vanity enough to look upon as a very neat one: Your Method of Investigating ye Locus of ye point *P* pleases me also because it is y^e^ only way by which such properties can be discovered; But before I proceed to prove that y^e^ Equation you bring out belongs to a Circle, it may not be amiss to lay down something more general on that Head. In order to which let y^e^ general Equation be Proposed; then I affirm

1. that if $r = 0$	the	a parabola.
2. that if $r = 1$, and its sign be $-$	Curve	a Circle.
3. that if $r >$ or < 1, and its sign be $-$	will	an Ellipsis.
4. that if y^e^ sign of r be $+$	be	an Hyperbola.

which four cases, for y^e^ sake of perspecuity I shall demonstrate separately. . . . "

After stating in his next communication to Blake, "I have defered writing this long in hopes of first having your Opinion of my last and y^e^ Difficulties therein that may require to be obviated; but beginning to suspect that your knowledge of my ill health may have been y^e^ chief cause that may have prevented my having heard from you," Simpson gives an explanation of an equation with which Blake had found difficulty.

"The Difficulty which you desire to have explained is, *how I obtain y^e^ Equation*

$$Ax^{n+1} + Bx^n + Cx^{n-1} \text{ \&c.} = 1^n + 2^n + 3^n \ldots x^n;$$

But y^e^ answer is easy since it is got at once by a bare supposition or postulatum. But you will ask why a series of that form rather than another, as for instance, $Qx^{n+2} + Ax^{n+1} + Bx^n$ &c. should be assumed. I say for let y^e^ assumed series be what it will, you never can in any case bring out a false conclusion therefrom, and y^e^ only reason why $Ax^{n+1} + Bx^n + Cx^{n-1}$ &c. is chosen rather than $Qx^{n+2} + Ax^{n+1} + Bx^n$ &c. is, because it appears from y^e^ Nature of y^e^ problem that y^e^ former of these

Series will be affected with y^e^ fewest Imaginary Terms; and therefore require y^e^ Least Trouble in y^e^ Operation. . . . "

The next two communications from Simpson contain an explanation of "y^e^ cause of y^e^ precession of ye Equinoxes," and of "a new solution to Kepler's Problem." Meanwhile, however, Blake had worked out a plan for an engine concerning which he desired Simpson's opinion, and in regard to which the latter wrote:

"A new method for exhausting a Receiver &c, could it be done without ye Help of Fire, Pumping, &c would, undoubtedly, be of service; But as I am of Opinion that there can be no way of effecting it, or making water to ascend, without removing a Weight equal to ye pressure of ye incumbent Air, I despair of seeing anything better for that purpose than what is already in use. I do not say ye Thing is absolutely impossible, yet 'till you are pleased to communicate your Sentiments and Contrivance I must reason to be of a different opinion.

"As to ye demonstration of what I communicated to you in relation to Kepler's Problem, I must beg leave to refer you to a small Tract of mine going to be published under ye Title of *Mathematical Essays.* These Essays were drawn up since I had ye Honour of your correspondence, and some of them from your particular Intimation, and therefore have a more immediate claim to your Patronage than the Book of Chances. Upon this presumption, I drew up ye inclosed Address, which I humbly hope you will not disapprove; but if you will not grant Leave for anything more than a bare inscription or had rather take ye Chances[11] into your protection I shall with equal Thankfulness accept ye Honour, tho' indeed it will give me a little concern to be denyed ye use of an Opportunity of expressing my Gratitude and ye great Esteem I have for your Friendship. . . .

"Both these Tracts, I expect, will be published in about 2 months. I wait with impatience to see what Farther Advances you have made upon Fluxions, and am,

"Honoured Sir. . . . "

It was in the process of elucidating the first and second books of the *Principia* in correspondence with Blake, that this volume of essays previously mentioned, suggested itself to Simpson, and

[11] *The Nature and Laws of Chance*, London, 1740.

was published by him under the title of *Essays on several curious and useful Subjects, in speculative and mixed Mathematicks; in which the most difficult Problems of the first and second Books of Newton's Principia are explained.*

Henceforth more time was devoted to the perfection of Blake's proposed *Essay on Fluxions*, in connection with which Simpson wrote the following criticism:

"Honoured Sir,

"As I don't think it will be of any great Importance to ye Subject, to know when a progression of Fluxions is infinite and when limited, and as I would, as much as possible, avoid discouraging a Learner with difficult Demonstrations in y^e^ Beginning of y^e^ Work; I should in y^e^ illustration just observe when y^e^ progression is limited and when not; because y^e^ Reasons of y^e^ Thing will better appear from what follows in y^e^ general Rule; for if it be evident from what is there proved, that y^e^ Index n, $n - 1$, $n - 2$, &c. at every opperation or succeeding Fluxion is decreased by unity it is equally evident that y^e^ progression only be limited when n is a whole positive number—I like your distinction extreamly *that the Progression is finite only when y^e^ last Differences of y^e^ Forces are equal to nothing.* . . .

"I communicated to Dr. Bevis that part of the letter which related to him. He gave me the following observations of ye Comet (taken by him at Mr. Graham's). . .

"The conclusions were taken from a geometrical construction and therefore cannot be quite as exact as those derived from a numerical calculation; besides ye Doctor has since informed me that there was a mistake of about 1/20 in ye time of ye last Observations which may possibly cause an error of 2 or 3 Degrees on some of ye foregoing numbers, but this is ye best account I am able to give of ye Comet at present. . . . "

During the next two weeks Blake favored Simpson with an account of his method of considering the fluxion of a triangle which he later included in the essay.[12] For the proposed lemma, "The Fluxion of the Area *ABC*, whether triangular or curvilineal, is

[12] Ibid., p. 8 et seq.; see also Cajori, *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse*, Open Court Publishing Company, Chicago and London, 1919, p. 173.

the Rectangle $\dot{x}y$," Blake offers the following illustration by way of explanation:

Suppose B to move along AF while the ordinate y terminates in the curve AC; "And, at any proposed Position BC, conceive y to become constant," while B "moves uniformly any constant Time, mn, with the Velocity at B, over the Distance $\dot{x}$ or BD; for then will y in the Time mn uniformly generate the Rectangle $\dot{x}y$, which Rectangle is plainly the Fluxion of ABC in this Position (*per Definit.*)."

To this illustration Blake added the following scholium:

"It has been commonly objected to the Accuracy of Fluxions that the Trapezium or curvilineal Space $BCdeD$, not the Rectangle

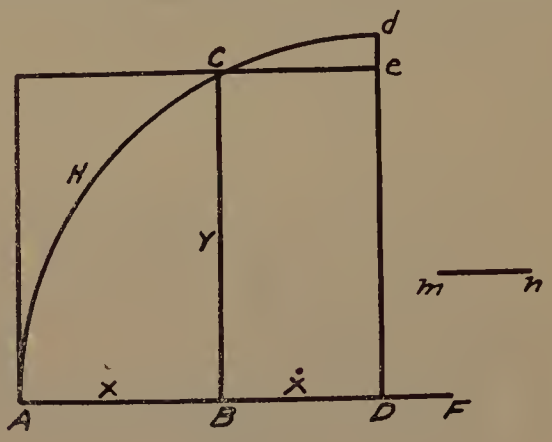


Fig. 8

$\dot{x}y$, is the Fluxion geometrically exact. But, this Objection is built, I apprehend, upon a false Idea of the Thing. It supposes a Fluxion a complete Part of a flowing Quantity, and an Infinity of Fluxions to constitute the flowing Quantity, which are Mistakes (*per* Definition and Lemma) . . . if $\dot{x}$ be imagined infinitely little, an Infinity of Increments may constitute the Area ABC. But, in Fluxions, our Reasoning is quite Different: a Fluxion can no more be called a Part of the Fluent, than an Effect a Part of the Cause. For Instance; from the Fluxion given we know the Fluent and *vice versa*, just as when a Cause is known to produce a certain Effect, we can infer the one from a Knowledge of the Other. . . . "

This letter induced Simpson to some independent thinking upon the subject, which he delighted in communicating to Blake:

"Honoured Sir,

"Since ye Favour of your Last I have reperused the whole of what has been said upon ye Head of Fluxions according to your last definition with a great deal of care and attention, and rest abundantly well satisfied both with regard to ye Method itself and ye Manner in which you deliver it; and am extreamly glad that you are now able to effect your Desires without borrowing aught from ye Arithmetic of Infinites, which, you know, I never was fond of, being well assured that to render it accurately true and, at ye same time, sufficiently plain and intelligible, would be a Difficulty vastly greater than any in Fluxions considered in y^e^ Light in which you now have put them. What you add in relation to y^e^ Inverse Method is perfectly right; for there can be no finding of Fluents, without a previous Knowledge of Fluxions, nor could we know that x^3 is y^e^ Fluent of $3x^2\dot{x}$, without knowing first, from y^e^ Direct-Method that $3x^2\dot{x}$ is y^e^ Fluxion of x^3.

"As to your solution to my last problem, it is exceeding short and elegant, and I must assure you, many times preferable to my own. I would not willingly, by proposing problems, interrupt your compleating what I wish to see effected, I mean, y^e^ Result of your Thoughts upon Fluxions; but since you desire something this way may be continued, please to take y^e^ following:

"Problem:

"Suppose *ECGHDFE* to be a Cask, full of Liquor, formed out of y^e^ given Spheroid *AB*, and of such a Length, *RS*, that it shall

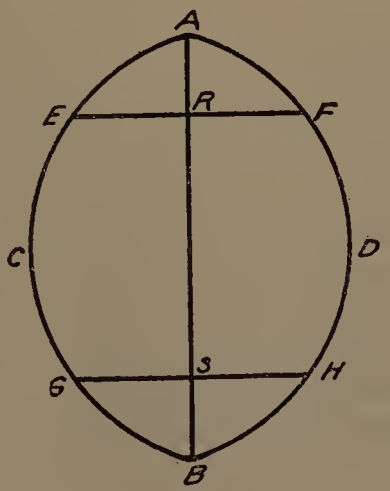


FIG. 9

require more Time to empty it self, out a given Hole in its bottom *GH*, than any other Cask that can possibly be formed of ye same Spheroid; 'Tis required to find ye Length of ye Cask on ye Ratio of *RS* to *AB*.

"N! B. The Top and Bottom Diameters of ye Cask are supposed to be equal.

"Ruminating upon your Manner of considering ye Fluxion of a Triangle, a Thought occur'd, which as it then much pleased me; I shall, without Alteration, take y^e^ Liberty to lay now before you tho' you will easily see it is y^e^ same in effect with what you have since favoured me with in relation to y^e^ Fluxion of a Rectangle.

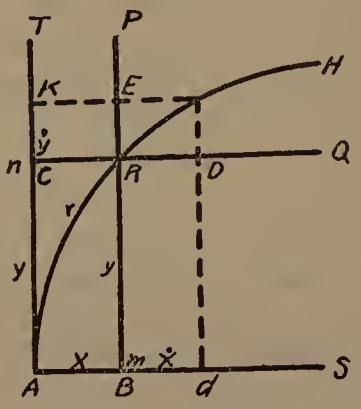


Fig. 10

"Suppose two Bodies *B* and *C* to move from *A*, at y^e^ same Time, Towards *S* and *T*, and to carry along with them ye perpendicular Right Lines *BP* & *CQ*, so that, by means of these Lines, a Rectangle *ABRCA* may be generated. Let ye Curve or path described by ye Intersection of ye said Lines (which varies according to ye Velocities of ye Bodies) be represented by *ArR;* and let *md* & *nK* be ye uniform Spaces that [the Bodies] would be describ'd, in any given part of Time, with ye Velocities at any assigned cotemporary positions *m* & *n*, that is, let *md* be y^e^ Fluxion of *Ab*, and *nK* y^e^ Fluxion of *AC*, according to your Definition, supposing y^e^ Flowing quantities *AB* and *AC* to be denoted by *x* and *y* respectively: Then, from what you have layd down with regard to y^e^ Fluxion of a Triangle, I think it appears that $y\dot{x}$ and $x\dot{y}$ must be y^e^ Fluxions of y^e^ spaces *ABRrA* and *ACRrA;* because what you say there holds equally here, and

may be apply'd with y^e^ same Propriety to these or any other curvilinear Spaces: But these two Spaces taken together are always equal to, and make up y^e^ Rectangle, *BC*, therefore, $y\dot{x} + x\dot{y}$, the sum of their Fluxions must consequently be y^e^ Fluxion of that Rectangle. Q. E. I.

"I call'd upon Innys y^e^ Bookseller, and he wanted to know how to send your Books to you.

"I likewise called y^e^ Doctor, and he told me he was about to write to you, and had defer'd it so long on Account of something relating to your Instruments; the particulars of which he will better inform you. Mr. Story acquainted me that ye Doctor had been with him and desired that he would not send down ye Tellescope 'till he had wrote to you about it; when he saw your Letter he was at a Loss what to do in ye Affair, but fearing, from what ye Doctor had said to him that something in it might be imperfect, concluded it best not to send it 'till ye Doctor, or I, had some further Advice from you about it. . . .

"My Book, I am now pretty positive, will be published in less than a Month. . . . "

Blake spoke of the book of *Essays* in a letter to Simpson written January 9, 1740, while at the same time communicating his thoughts on the "higher order of fluxions." He writes:

"Dear Sir,

"I now take an Opportunity to thank you for your Book, and beg your Acceptance of what you'll find on the other half sheet, as a Token of my Gratitude and Esteem. The Book entertains me prodigiously.

"I doubt I shall quite tire you with nonsense about Fluxions. What is said to illustrate a succession of them, I suspect foreign to the Purpose, and in Reality darker than the Thing itself. The Method I once hinted to you in a Letter from Bath, is perhaps, more elegant and clear than any other I can contrive; & therefore it may be worthwhile to consider it again viz. The Fluxion of a Body moving in *AB* with a Velocity as the Differences betwixt the first Fluxions, of any Fluent, taken continually, is the 2d. Fluxion of that Fluent. And the Fluxion of a Body moving in *CD* with a Velocity as the Differences of these Differences is the 3d. Fluxion of the said Fluent. And again, the

Fluxion of *EF* generated with a Velocity as the Differences of these last Differences, is the 4th Fluxion; and so on 'till the Difference = 0. For instance, the Fluent $\frac{xx}{2}$ is a Triangle and the difference betwixt y^{e} first Fluxions is always 1. Therefore *AB* is generated uniformly, and *CD* not at all. In the conic Parabola $\frac{2x^3}{3}$, *AB* will be generated with a Velocity accelerated as the Numbers 3.5.7.9. &c, *CD* uniformly as 2.2.2. &c and *EF* not at all.—*Thus I think we shall best exclude infinitely small and Infinitesimal Quantities; the Bane of Fluxions.*[13]

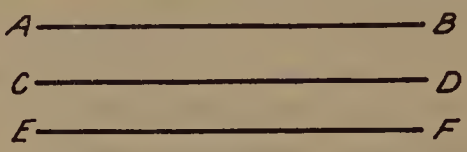


Fig. 11

"Besides, by this Method I am enabled to conceive very distinctly all the Orders beginning at once to be generated; which is no inconsiderable Advantage, for I'm persuaded Nothing has conduced more to jumble Fluxions and the Arithmetic of Infinits together than the supposing a successive Generation of the Former. . . .

"If Doctor Bevis does not write to me very soon, I will pursuant to your advice, come to some positive Resolution about the Instruments. He is an odd, rattling Creature as he ever was.—Pray be a most impartial and severe Critic of what is or may be sent you abt. Flux. by

"Your obliged Friend and
"Servant,
"Francis Blake."

"Herrington, Jany. 9, 1740."

Evidently Simpson was not wholly in sympathy with Blake's proposed method of treating the "higher orders of fluxions." In reply he proceeded to outline his conception of how it should be done; in the course of which discussion he states as a corollary:

[13] This statement shows the influence of Berkeley's *The Analyst*, 1734; for this is one of the objections to fluxions cited by Berkeley in that work.

"Hence it appears that ye First Fluxions of Quantities are always as ye Velocities with which those Quantities are Generated, and ye 2d. Fluxions as ye Increase or Decrease of such Velocities. It likewise appears that by 2d, 3d, and 4th &c Fluxions, are meant such Fluxions whose Fluents are themselves Fluxions to other proposed quantities, and ye Manner of considering and determining all such Fluxions is ye same manner as though they were first Fluxions, they being actually such to ye Quantities from whence they are immediately derived.

"The particulars duly weighed will (I hope) remove all ye difficulties and seeming Inconsistencies so often complained of in the progression of Fluxions. There is no essential difference amongst them; the process is only ye more difficult the higher we go. . . "

He continued his explanation by suggesting a lemma and proposition which appeared to him desirable to be incorporated in the proposed Essay, and he closed with a remark on Blake's own proposals concerning its contents:

"Thus far I have judged it necessary to transcribe. What remains behind in ye succeeding Scholium, proposition, and its corollaries seems to be so well weighed that I cannot pretend to hint at any Alterations for ye better, and therefore shall now conclude, with subscribing myself, . . .

"I cannot help leting you know that I am pleased with your Introduction or Apology; it is the most genteel, modest, and expressive of any thing I have seen on ye like Occasions."

Judging from what Blake has to say in his next communication to Simpson, February, 1740/1, he is able to reconcile Simpson's ideas with his own:

"Dear Sir,

"You are very good to be so punctuall in answering my Letter; and I can assure you there is no more effectual Way upon Earth of obliging me, than to speak your Sentiments with Impartiality and Freedom. The least Tenderness at this Time would be ye utmost Cruelty.

"As to the higher Orders of Fluxions, I'm persuaded we Both meant alike; but what perplexed me was the value of *EF* when *CD* is uniformly generated, as though it was esteem'd a Part of the *EF* wch. is the Fluxion of variable *CD* and Fluent of *GH*.

That proceeded, no Doubt, from a wrong Conception; and now, I perceive y^t *F* is permanent or may be permanent at *E*, when the Fluxion of *CD* uniform, is consider'd; and in every other case the Point *F* is in motion, which must necessarily give Rise to another Fluxion &c. But still I cannot help thinking there wd. be an Advantage in shewing y^t *EF*, consider'd as a Fluent is generated by the successive Velocitys of *D* computed as if ye Motion began in every point of ye Line *CD*, and so in like manner *GH*, by the Velocity of *F* beginning to move from every Point of *EF*. I would illustrate it thus, viz *CD* is a Space described by an Uniform Gravity (for Instance) in 7″. The first Fluxion *EF*, or uniform Space might be describ'd in (*mn*) 1″ with ye Amount of Velocity at *D* as 224^f; but that is equal to 7×32. Now 32^f is the Effect of the Force of Gravity in 1″ always, and 7″ is the Time of *CD*'s Description, ergo *EF* is generated together with *CD* in that Sort, and is also the uniform Space which might be generated in *mn* with ye Amount of Velocity at *D*. Again, if the Force of Gravity be unequal, *EF* is then manifestly generated with a variable Velocity, and this Variation in *EF* will generate *GH*, as *EF* was before generated by *CD;* wch I think seems to prove the Truth of the 2d. Illustration in my last Letter, & the consequent Observation about y^e Instantaneous Generation of the several Orders. Indeed the Equation $AB \times v = mn \times V$ is only true in the Case of an uniform Force, since otherwise *v* is variable as often as *AB* contains *mn*, and therefore the Aggregate of these $= mn \times V = EF$ &c.

"But should not we wait, think you, for a Sight of Mr. McLaurin's Work, which you say he is about publishing? I have no Apprehension that he will interfere with me for the Reason given in my Introduction; besides I remember to have seen several Sheets of it, long ago, and if he has not altered his method, I dare say he addresses himself only to great Mathematicians. He begins with a tedious Account of Archimedes's Manner of Reasoning in Mathematics; after which he lays down a vast Nr. of Lemmas concerning the Nature of Motion; and the Method of Demonstration used throughout is (as well as I then cd. judge) an exact Copy of the Ancients.

"I shall observe your Commands about the Conclusion. But I'm sure what is there said of you could neither argue me a

Flatterer nor an injudicious Man; whatever else the World might maliciously insinuate.—You'll be pleas'd to acquaint me whether or no I am yet right as to the higher Orders of Fluxions.

I am,

"Your obliged Friend and
"Humble Servant,
"Francis Blake."

"West Herrington Feby. 8, 1740/1."

"The Fluxion of $m - 1 \times mx^{m-2}\,\dot{x}^{2}\,mx^{m-1}\,\ddot{x}$ is $m - 2 \times m - 1 \times mx^{m-3}\,\dot{x}^{3} + m - 1 \times mx^{m-2}\,\ddot{x}\dot{x} + mx^{m-1}\,\ddot{x}$, I think. Did you ever apply Fluxions to Sr. Isaac Newton's Doctrine of Sounds? I find some Difficultys in it."

Maclaurin had corresponded with James Stirling as early as 1734 concerning this treatise to which Blake refers, when he wrote of what he proposed to include in his work:

"Upon more consideration I did not think it best to write an answer to Dean Berkeley but to write a treatise of fluxions which might answer the purpose and be useful to my scholars. I intend that it shall be laid before you as soon as I shall send two or three sheets more of it to Mr. Warrender that I may have your judgments of it with all openness & liberty. This favour I am rather obliged to ask of you that I had no body to examine it here before I sent it up on whose judgment I could perfectly depend. Robt. Simpson is lazy you know and perhaps has not considered that subject so much as some others. But I can entirely depend on your judgment. I am not at present inclined to put my name to it. Amongst other reasons there is one that in my writings in my younger years I have not perhaps come up to that accuracy which I may seem to require here. When I was very young I was an admirer too of infinites;[14] and it was Fontenelle's piece that gave me a disgust of them or at least confirmed it together with reading some of the ancients more carefully than I had done in my younger years. I have some thoughts in order to make this little treatise more compleat to endeavour to make some of Mr. De Moivre's theorems more easy which I hope he will not take amiss as I intend to name everybody without naming myself. . . .[15]

[14] One of the objections Berkeley brought against fluxions was the use of infinitely small quantities.

[15] Yet De Moivre did object to Simpson's efforts along this line.

What Blake has to say of Maclaurin's work is not essentially different from what is said of it today. It is interesting that Maclaurin's manuscript was in circulation among his students, and through them among others, as Maclaurin's testimony in a letter to his friend Stirling would show.[16] It is barely possible that Maclaurin's work was as great a single influence in the development of Blake's *Essay*, as was the advice of Simpson. Later correspondence seems to corroborate this theory.

Meanwhile in his next letter Simpson admits the similarity between his ideas and those of Blake "concerning the higher orders of Fluxions," although he writes:

"I see now that your way of considering ye higher Orders of Fluxions differs but very little from what I proposed in my last and am sorry that I should interrupt the Maturity of your Invention: I must, however, persist that ye Objection which you bring to my Manner of considering ye Fluxion *EF*, is without sufficient Foundation, and wholly grounded on a wrong Apprehension of my true Meaning; for I do not consider *EF* as actually generated by an uniform Velocity but as ye Quantity that *might be* generated &c because there is no Manner of Necessity, neither is it rèquired by the Definition, that *EF* or any other Fluxion should be actually described with an uniform Motion. . . . I would not be thought to urge this in Vindication of my own Judgment but as what appears to me ye most proper and intelligible; for I scruple not, and think myself in justice, obliged, to acknowledge that *whatever Hints I may have advanced with regard to these higher Orders of Fluxions were chiefly derived from a former Letter of yours, to which I stand indebted for a more general and less complex Idea on the subject than I before entertained.*

"As for the practical rules necessary for determining all orders of fluxions the 3 common Ones for a Rectangle, power, and a Fraction, will (In my Opinion) be sufficient, . . .

"It may not be amiss however to give some instances of 2d fluxion &c among ye Examples succeeding ye general Rules and as to an Illustration by Gravity I apprehend it will be sufficient to

[16] See Correspondence of Maclaurin with Stirling, found in Tweedie's work entitled, *James Stirling, A Sketch of His Life and Writings Along with His Scientific Correspondence*, Oxford Clarendon Press, London, 1922, p. 57 et seq.; letter quoted above from pp. 73, 74 of this work.

shew (or to take Notice) from thence that there is a progression of Fluxions, arising from Causes actually existing in Nature; and this I fear is indeed all that can be actually done herein to be of any real Use; because to investigate ye higher Orders of Fluxions according to ye known Law of Attraction of our system (or even any other except where the Force is uniform) will be a Matter of insuperable Difficulty, there being no way to come at, even, the first Fluxion without compound surds of ye Quadrature of Curves. . . . "

It was Francis Blake's wish to include in his *Essay* an "illustration by gravity," to which Simpson referred in this last communication. This illustration he intended to follow in order after the three common rules in the finished work.

In a letter sent February, 1740/1 to Simpson, Blake also sets forth the conclusions he intended to include in the proposed *Essay:*

"To proceed farther would naturally engage me in the Application of this doctrine to Mathematics & physics; which is quite foreign to my present purpose, and indeed little suited to my slender skill and experience in these Studys. I might perhaps with justice enought add too, a General Performance of that kind is little wanted; for what more elegant examples and solutions can we expect than those in the works of our Mathematicians & particularly those of my ingenious friend Mr. ——[17] There also, the theory is most concisely explained and demonstrated in a way agreeable to his fine imagination; but that could be no common measure of men's perceptions; & therefore this plain essay may have its use. Whether something still plainer be even yet wanting or not, I shall leave to the Readers determination. . . . "

In the postscript Blake has written, "If you think sincerely it will do, I shall send you a fair Draught with a letter to *Innys* to print off a few copys presently (*Anonymous tho' till I see the success*) for it would vex one to be forestalled after taking so much pains; and some may hit upon a method to frustrate this, tho' I believe no one can readily stumble upon this method so very much out of the high road. You'll observe *I have been forced*

[17] Mr. Simpson, no doubt.

to change the style for the worse in order to preserve an Uniformity: but in Mathematical pieces Style is not, nor ought to be regarded. . . "

The introduction that Blake had in mind at this time for the *Essay* was an especially modest one. He was attempting to reach the class of mathematicians to which he himself belonged. His aim was to simplify the subject of *fluxions* and bring it down to the level of beginners. The preface and introduction were included in a letter Blake wrote to Simpson, February 15, 1740/1. The preface which he here proposes gives the best indication as to the nature of the *Essay*, and is essentially the same as the one finally incorporated in the finished production. It reads as follows:

"To The Reader:

"The many Disputes concerning Sr. Isaac Newton's Method of Fluxions induced me to try, if that most useful & noble Kind of Investigation might not be establish'd upon more obvious Principles. This gave Birth to the following Essay; which, therefore, you are desired to consider as an Illustration of the Doctrine, & not of Sr. Isaac's Manner of delivering it: about that I neither mean nor pretend to take Part in any Dispute. It was, doubtless, agreeable to our great Author's unbounded Invention and Discernment; but, I presume, a more familiar Demonstration and Phrase will neither be unacceptable to you, nor at all derogatory to the Merit of his Performance, whilst they tend to confirm and elucidate the very same Truths.

"An Essay on the Theory of Fluxions.

"The Productions of an exalted Genius are very liable to Misconstruction and Cavil, as the Subject is often clouded with some natural Intricacy. Hence particular Illustrations and plainer Methods of Proof become requisite; but it is a Truth not enough attended perhaps, y^{t} these Devices are not the proper Task of ablest Pens. Such Enquirers, too nearly resembling the Author, never feel all the Wgt. of a pressing Difficulty: besides, a quick Conception is still apt to cause an unusual Conciseness; which, no Doubt, must obscure the Sense to many who perceive not but by an easy chain of Consequences.

"This seems a reasonable Apology for my attempting to il-

lustrate the Theory of Fluxions, that important Doctrine. What I can offer may be better adapted to give general Satisfaction, than are the lofty Essays of eminent Mathematicians. Such however is my Encouragement to the great Undertaking, & so are my Hopes of Success founded; if I should fail, In magnis voluisse, sat est.[18]

"General Definition.

"The Word Fluxion, properly apply'd, always supposes the Generation of some Quantity (term'd Fluent or flowing Quantity) with an equable, accelerated or a retarded Velocity, and is Itself the Quantity which might uniformly be generated in a constant Portion of Time, with the Amount or Remainder of that Velocity at the Instant of finding such Fluxion.[19]

"Illustration.

"Suppose a Point *D* to move from *A* with any Velocity viz equable, accelerated, or retarded. And at the Instant *D* leaves *A*, let *EF* be taken equal to the Distance which might be uniformly generated in a const^{t}.—Portion of Time *mn* with the Velocity of *D* at *A;* for then is *EF* the Fluxion of *A*.—Now if *AD* be generated with an equable Velocity, or in other Words, if the Velocity at *A* be the Velocity in every point of *AD*, it is plain *EF* is a constant Qty, and, of Course, can have no Fluxion. But on the other Hand, when *AD* is generated with an accelerated or retarded Velocity, let the Line *EF* be so increas'd or decreas'd by the Motion of a Point *F*, whilst *D* moves from *A*, y^{t} this increasing or decreasing *EF* may always equal the Distance which might be uniformly describ'd in ye constant Portion of Time *mn*, with the Amount or Rema^{r}. of Velocity at *D;* for thus is *EF* still the Fluxion of *AD* & Definition, and a variable Quantity too producing its' proper Fluxion *GH*. This is determin'd mutatis

[18] This introduction shows the influence of the controversies which began soon after the publication of *The Analyst;* the main disputes being, Berkeley's controversy with Jurin and Walton, and Jurin's controversy with Robins and Pemberton.

[19] This definition differs in wording from that contemplated in an earlier letter from Blake to Simpson, dated May 8, 1739.

mutandis as *EF* was at first viz take *GH* equal to the Distance which might be uniformly generated in the said constant Portion of Time *mn*, with the Velocity of *F*, whereby *EF* was increas'd or decreas'd as *D* left *A*. It is likewise called the 2d. Fluxion of *AD*, being the Fluxion to a Quantity that is itself the Fluxion of *AD*. Again, if the Original or given motion wherewith *AD* is generated be such y^{t} *GH* is also variable, let *IK* be taken equal to the uniform Space wch. might be generated with the Velocity of *H*, tending to increase or decrease *GH*, whilst *F* began to increase or decrease *EF* upon *D*'s leaving *A;* for so, *IK* is the 1st. Flux^{n}. of *GH*, 2^{d}. of *EF*, & 3^{d}. Flux^{n}. of *AD*.—

"And lastly, it is not difficult to conceive such an Acceleration in *D* y^{t} neither *EF*, *GH*, *IK* nor any other succeeding Quantity shall be uniformly increased or diminish'd by the Motion of these regulating Points, *F*, *H*, *K*, &c, whence there will most manifestly arise a Progression of Fluxions in Infinitum. The Descent of heavy Bodys, accurately considered, comprehends this Infinite Progression since in the true Theory of Gravity the Force is vary'd every Moment.

"And here perhaps it may not be amiss, briefly to exemplify first and 2^{d}. Fluxions by the Effect of an uniform Gravity, as it will also serve further to explain how the Succession is limitted & may be continued. . . .

"Dear Sir, If this Illustration be right, I perfectly understand what I am about, and therefore as soon as you favour me with a Line, shall put the finishing Hand to the Essay. What I drove at all along was to have *EF*, *GH*, &c. generated together with *AD*, not considering y^{t} the Force at *A* must necessarily cause a Fluxion at y^{t} Point as well as any other, & therefore oblige me to determine a prior Value of *EF*. In the other Case the Velocity of *D* must have increas'd from nothing.—I'm tired to Death with writing, as no doubt you are with Reading; so I remain,

"Dear Sir,

"Your Sincere Friend and Humble

"Servant,

"Francis Blake."

"West Herrington, Feby. 15. 1740/1."

"P. S. The Illustration is ill expressed but if that be the only Fault I can easily correct it.—"

It was in this manner that the *Essay* destined to have a profound influence on the later development of the subject of fluxions reached completion. As has already been stated, Simpson was influenced by the *Essay* in the 1750 edition of his *Doctrine of Fluxions*,—as is indicated by his acknowledgment in the preface of indebtedness to an unnamed friend for the determination of the fluxion of a rectangle. Of this Sir William Rowan Hamilton commented, some years later:

"I notice that Thomas Simpson treats fluxions as *finite*. . . . Thomas Simpson's conceptions appear to have been very clear and distinct and I do not venture to say that the geometrical investigation which he gives of the *fluxion* of a rectangle, avowedly supplied to him by a young but unnamed friend, is *insufficient* in itself, but it fails to convince me, perhaps because I was not early accus-

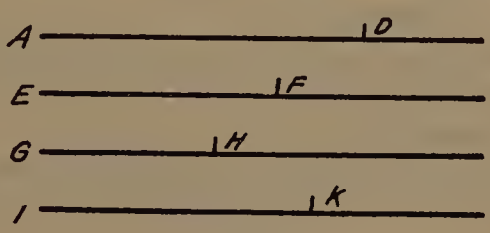


FIG. 12

tomed to fluxions. Certainly there is no *neglecting* of *ab* or $\dot{x}\dot{y}$ as *small;* for in the fact that *rectangle of* the *fluxions* is not represented at all in his Figure. . . . He conceives the *varying rectangle xy* to be the *sum of two mixtilinear triangles*, of which the *two separate* fluxions are $y\dot{x}$ and $x\dot{y}$. This is very ingenious, but I do not feel sure to what degree I could *rely* on it and build upon it any superstructure, if I were now coming, for the first time, as a *learner*, to the subject. However, I suppose that a pupil, if reasonably modest or even prudent, will take, *for a while*, his teacher's statements upon trust; reserving to himself to *return* upon them, and to examine closely their truth and logic when he shall have acquired some degree of familiarity with the subject taught."[20]

Simpson writes on May 5, 1741, of a contemporary view taken

[20] *Life of Sir William Rowan Hamilton*, by H. P. Graves, 1882, vol. iii, p. 571.

of the essay while at the same time giving an account of the changes he has made in Blake's proof while superintending its preparation for the press:

"I shall desire to decline giving my Opinion of ye Performance as one that I may perhaps seem too much prejudiced in its Favour, but must acquaint you that having shewed it to two or three very considerable Judges, without taking any notice of my acquaintance with ye author, they were all pleas'd to express their Approbation of it very much affirming that it contained a clearer Notion of Fluxions than anything they had seen Extant, which I will assure you, was no small pleasure to, . . . Honoured Sir, Your most obliged and most Obedient Servant, T. Simpson."

The *Essay* does not seem to have had wide circulation, for in a postscript to a letter Simpson wrote to Blake in November, 1741, occurs the following:

"All that I have heard speak about your Essay seem extreamly well pleas'd with it; and I heartily wish it was in more Hands (were better known)."

Blake himself was evidently well satisfied, for he even contemplated a reprint, as witness a letter written on March 2, 1741:

"Dear Sir,

"Should my Essay be reprinted, the Plan I propose to observe, if you approve of it, is as follows viz.

"1st: To abate a little of my Modesty in the Introduction.

"2dly: To inlarge the Illustration, thus, vide Page 5, after the Words, *rendering GH constant.* And, tho' the Impulses on *D* sh^d^. vary, whence it w^d^. not be uniformly accelerated, ye Limitation is likewise evident where they vary by a common Difference. Thus when the Arith^l^. Progression 1.2.3.4.5.6. &c expounds the accelerated Forces; 'tis plain *F* w^d^. be uniformly accelerated, the Velocity of *H* uniform, & *IK* w^d^. be a constant Qty. Or suppose 1.3.5.7.9. &c to expound these Forces; then *F* w^d^. be accelerated like a Body impell'd by the Law of ye first Progression, *H* uniformly accelerated, the Velocity of *K* uniform & *LM* w^d^. be a constant Qty. But, on the other Hand, if the Squares of ye natural Nrs. 1.2.3.4.5.6. &c viz 1.4.9.16.25.36. &c are to expound ye variable Impulses, the Progression becomes infinite. For, here ye Difference flows; & consequently no Fluxion

can ever be uniformly increased or diminish'd by the Motion of its' regulating Point *F.H.*, *K*, &c. And 'tis only changing the Centrifugal Force of *D* into a Centripetal One, to make this Case the Descent of heavy Bodys according to the true Theory of Gravity above mention'd.

"3rdly: To deduce the Fluxion of Log^{c}. & Expon^{l}. Qtys from the Principles deliver'd; & insert them at Page 16. after Line ye 5th:

"4thly: To shew the Advantages of this Manner of conceiving Fluxions, in that the Idea of Motion is always kept up in solving Problems, & the whole Process becomes thence scientific.

"5thly: Some Instances thereof in Quadratures, Rectifications, & Physical Problems.

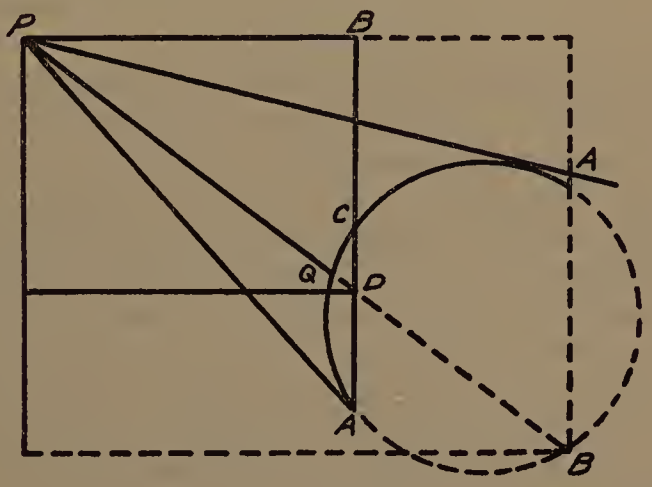


Fig. 13

"6thly: Conclusions, little vary'd.

"I believe your last Problem may be constructed from the Solution of a former you set me; where the Diagonal of a Square is proved to be the Line of quickest Descent from a Point to a given vertical Line A:B:—Now, from *P*, draw *PA* a Tangent to the Circle, erect *AB* ⊥ to the Horizon; and the Segment *AA* is the Limit. But *P* will arrive at *D* sooner than at *C* or any other p^{t}. of *AB;* therefore it seems as if that also was the required Plane for the Circle.—If this will not do, the Value of *PQ* must depend upon the Properties of the Circle, and the Expression

for the Time of Descent be made a Minimum, as in the other Case.

"You don't mention my Quadrant, which was left at Story's with the Telescope. I thank you for the Trouble you have had in calling there & so often at Mr. Gibson's; and am on many Accompts

"Dear Sir,
"Your Obliged Friend &
"Humble Servant
"Francis Blake"

"Herrington, March 2. 1741."

"P. S. Don't you think some Prob: abt. the Descent of Bodys will be the aptest Application of my Method to Physicks?"

The influence of Thomas Simpson upon Blake throughout the development of this essay is self evident, which means that the influences mentioned as having operated on Simpson had in turn reacted on Blake. Simpson's ideas concerning *fluxions* before the development of this essay can be seen through a study of the 1737 edition of his *Treatise of Fluxions.* As has been previously remarked his sources of information at that time were had from Stone's translation (1730) of l'Hospital's, *Analyse des infiniment petits,* a treatise in itself inaccurate to the extent of superimposing Newtonian language upon a continental conception: as is seen in the translation of "différence" as "fluxion," and in the substitution of the Newtonian for the Leibnizian notation. Simpson also had read Muller, whose interpretation was of an entirely different nature.

In the course of his study under Simpson, Blake had also read Newton's *Methodus Fluxionum.* In addition, he acknowledged having known of Maclaurin's work, so that on the whole it is remarkable that he avoided the confusion felt by many of his contemporaries when being presented with *fluxions* defined as infinitesimals, velocities, and finite distances. Blake's essay gradually gained recognition, and as late as 1809, was reprinted with the fourth edition of John Rowe's *Doctrine of Fluxions.*[21]

[21] *An Introduction to the Doctrine of Fluxions, Revised by several Gentlemen well skill'd in the Mathematics.* Felicibus inde Ingeniis aperitur Iter-Claudian. London, MDCCXLI. See Cajori, *History of Fluxions,* pp. 175, 176.

The year 1740 had been an important one for both Simpson and Blake. While Simpson was busy publishing two treatises, and Blake was organizing his essay entitled *An Explanation of Fluxions, in a Short Essay on the Theory*,[22] other events contributed to make the year eventful. Blake was appointed to a post of honor which somewhat interfered with his mathematical studies, so that he wrote to Simpson:

"The taste of ambition and a public life is so bitter to me that 'tis my opinion I shall now more than ever relish the sweets of retirement and Study. I determine to prosecute mathematics with redoubled vigour. . . .

"Did you return me Brook Taylor's *Perspective?* If you did I have left it at Mr. Gibson's chambers in Lincoln's Inn. Tell Mr. *Innys* I have got the *Vol. of Memoirs* from Manby and ask him if I left any other books to his care. . . . "

In congratulating Blake upon his new appointment, Simpson gave an account of his own affairs:

"The enjoyment of a Post of Honour, and a Taste of Mathematical Knowledge, to one who views things in ye Light which you do, will not be found incompatible, and tho' I might, by your entering a publick life be in some measure deprived of ye pleasure I may have promised my self from an uninterrupted continuation of your studies, yet I have this to please me, ye same occasions which bring you to town will give me more frequent opportunities of the pleasure of your conversation. . . .

"The gentleman in Lincoln's Inn Square who I informed you was about to commence my pupil at your leaving town is a Member of Parliament for great Marlow in ye County of Bucks, and proves not only a very valuable Scholar, but has made a surprising progress (which I mention knowing that it will not be disagreeable to you to hear anything, that tends to my Credit or Advantage). Your book of *Perspective* I returned to you before you left Henrietta Street; . . . "

Although Blake continued his studies, the extent of his difficulties is perhaps best shown in a letter sent to Simpson, January 29, 1741:

[22] Printed for W. Innys, at the West-End of St. Paul's, London, MDCCXLI.

"Dear Sir,

"It is my Misfortune to be involv'd & hurry'd in Business, whilst at the same time I retain all the wonted Inclination for Study. This makes such a Jumble in my Head y^t I'm scarce fit for either. But, duce on you, I think you might grope out my Meaning a little better than to imagine I thought the Log. & Velocity of Log. the same. I meant y^t *BC* being the Expont. of *DE* & *BCl: a* the Log of *DE*, whatever is the Velocity ($a^y\dot{y}1:a$) of the Log. of *DE*, the Velocity of *BC* may be expounded thereby (former Diagram). However, not to dwell upon Obscurities, let me try if I cannot deduce the Fluxion of a^y from my Lemma in some more obvious manner, & no less new. . . . What I all along wanted was a Reason why the Log. of Root must necessarily make a Part of the Fluxion. But it is plain to me now y^t it is introduced by our assuming first the Fluxion of the Log. & then multiplying that Fluxion by the Quantity; for the Product is the Fluxion of that Quantity. . . . "

A new topic engaged Blake's attention for a time. He became interested in Simpson's ideas with regard to geometric loci, so that in July 21, 1741, he wrote to Simpson:

"Dear Sir,

"I am so little used to the Method of Reasoning by Composition, Division, Inversion, and Conversion of Ratios that it was with Difficulty I saw the Truth of your Theorem. I am upon that very Account glad, however, you propos'd it, and likewise as it gives me a fair Opportunity to turn my Enquirys upon Geometric Loci and the Construction of Equations, where I am more at Loss than in most other Parts of Mathematics.

"A Demonstration of the Theorem,

"The given Ratio is $CO:BC::AC:\overline{AC - BC}$. Then $AC:CO::AC - CB:CB$, and $AC + CO:CO::\overline{AC - CB} + CB:CB$, that is $AO:CO::AC:BC$, or $AO:AC::CO:BC$. Again $AO - AC:AC::CO - BC:BC$, that is $CO:BO::AC:BC$. But $CO = PO$ and therefore by these Proportions $AO.PO.BO$ are continual Proportionals, and the $\angle O$ being common, the triangle *PAO* is similar to triangle *PBO*. Therefore $AO:PO::AP:PB::AC:BC$. Q.E.D.

"But to investigate the Locus of P, let AC (a) + CB (b) = s; $BO = \dot{x}$; $PO = y$; then $AO = s + x$,

$\overline{AP^2} = s^2 + 2sx + x^2 + y^2$.

$\overline{BP^2} = x^2 + y^2$,

& y^e Question $s^2 + 2sx + x^2 + y^2 : x^2 + y^2 :: a^2 : b^2$

Therefore $b^2s^2 + 2b^2sx + b^2x^2 + b^2y^2 = x^2a^2 + y^2a^2$,

or $b^2s^2 + 2b^2sx + b^2x^2 + b^2y^2 - x^2a^2 - y^2a^2 = O$.

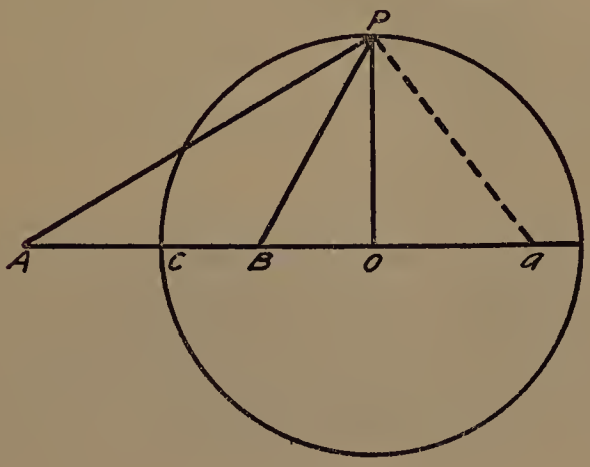


FIG. 14

"Now, Pray shew me that this Equation belongs to a Circle, and teach me as clearly as possible how to raise a general Expression from the Circle, by which all particular Cases may be examined. Our next step will include the Expressions relating to Conic Sections, I presume; but we cannot walk too gently in so rugged a Path.—

"I have nothing further to request at present, but your Acceptance of this Bill for five guineas from

"Dear Sir,

"Your sincere Friend &

"Humble Servant

"Francis Blake."

"West Herrington, July 21, 1741."

Simpson in the meanwhile began a study of conic sections, but met with difficulty, for he wrote:

"I could not 'till three days ago procure y^e *English Translation of Hospital's Conic Sections*,[23] and therefore have scarcely had time

[23] *An analytick treatise of Conic Sections: . . . made English by E. Stone.* London, 1723.

to fully acquaint myself with that author's Reasoning upon the particulars you Mention: I must however own, that, so far, I like his method very much and think it not only general, but, for ye most part, clear & intelligible; there are indeed some places that seem to me, a little obscure, and to want an explanation, and if these should appear so to you (as the thing upon ye whole is well worth ye Perusal) you may be assured I shall always be ready, and glad, to contribute my best Endeavours to remove any obsticle, and ever reckon it a great Happiness. . . . "

An interesting letter from Blake to Simpson, written November 22, 1741, shows the variety of Blake's mathematical inquiries at this time. He writes:

"Dear Sir,

"For the Benefit and Pleasure of writing to you again as soon as might be, I venture to send what first presented themselves as Solutions to your Problems; and since they may well stand in Need of an Apologie perhaps, I must do myself the Justice to say they occur'd almost at first Reading.

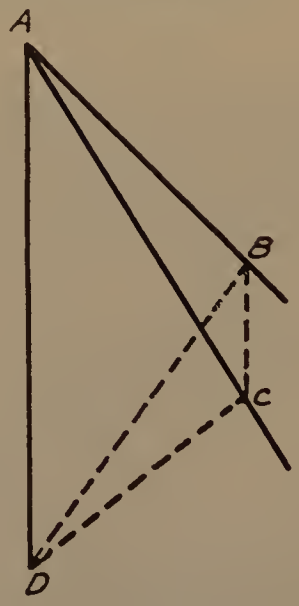


Fig. 15

"1st Prob:

"By Cor: 2 to Prob: 5 of Keil's Introd: at ver: Phys:, Perpendiculars from the Planes to *D* will give the cotemporary

Positions $B.C.$ of the Bodys, whatever be the Inclination of the Planes; & since the Prob; requires y^t BC be a vertical Line i.e. $// AD$, $\triangle ADB = \triangle ACD$. Therefore $\angle CAB$ is the Complement of the Angles $DAC + BDA$ $(= 2 \angle DAC)$ to a Right Angle. Q.E.I.

"Prob: 2.

"An Arch of a great Circle passing thro' the Ships when equally distant from the Port whence the Ship sail'd westward is, I think, their nearest Approach to each other; and this will happen when one ship has sail'd 3600 Miles west & the other 1800 miles towards the same Port, for then are they equally distant from it.

"I design soon to resume the Subject of Geo: Loci, & then you may be sure I shall look carefully to Hospital's Obscurities, and beg the Favour of your Assistance, if needful; tho' in the mean Time I sh^d. be more glad to see his Method handled by you, for probably that would clear up every Thing.—At present I'm engaged in Exponentials with the View mentioned in my last, and y^t if I succeed, they may be a proper Addition to the Essay, sh^d. it be worth while ever to take further notice of it.—Now, when I say x^v $1{:}x \times \dot{y} + yx^{v-1}\dot{x}$ is the Fluxion of x^v, I deduce it in some such Manner as this viz. By the Nature of Geo: Series every succeeding Term arises from the Multiplication of the last by a determinate Root, which here is $1{:}x$; and therefore x^v $1{:}x$ is a new x^v, & by the Property of Exponents this Involution is adding to the Expont. y, so when x is a fix'd Quantity, y only flows and the Fluxion is x^v $1{:}x$ (new x^v) $\times$ $\dot{y}$, agreeable to my Lemma. But if x flows also, then it is evident our new x^v, at the Instant of becoming so, is in the State of a common Power & consequently comprehended in the general Proposition whereby the Fluxion is $yx^{v-1}\dot{x}$; making the whole Fluxion of $x^v = x^v$ $1{:}x \times \dot{y} + yx^{v-1}\dot{x}$ as above.

"I observe y^e Logarithmic Curve is denoted by x^v, when x is supposed constant.

"Besides these Enquiries, I'm like wise very busy wth y^e Solar System, trying to make it obvious to myself how all the Irregularities arise from the mutual Attractions of Bodys according to the receiv'd Theory of Gravity. But Pray! if two Globes be placed in empty Space within the Spheres of their own Attrac-

tions only, and a projectile Force be impressed on one of them only, will not Both describe Orbits, & similar too, about their common Center of Gravity? I would demonstrate this from the Impossibility of that Center's being at Rest, or the Bodys keeping their due Distances from it, on any other Supposition—I am,

"Dear Sir,

"Your Real Friend & Hble Servt.

"Francis Blake."

"Herrington, Novr. 22, 1741.

"P. S.: If you please I will order your Book to be advertised at N: Castle, for Mr. Brysan thinks it will not go so well off otherwise."

The following month Blake again wrote Simpson concerning his thoughts on exponentials, having in the meantime thought of a new plan:

"Dear Sir,

"I am sorry to trouble you so soon again; but having Something to communicate by Way of Improvement to my last, & seemingly of Importance to the Speedy Maturity of my Design, I shall venture to trespass upon your Patience and will rely upon your Friendship for Pardon.

"Supposing, then, the Nature of ye Exponential Expression a^{v} to be well defined by a flowing area a^{v} as before described; in order to determine its' Fluxion, I proceed thus. Let the 1st Exponent of a be the Log: bb of a; the 2^{d}. Expo^{t}. of this a, or the 2^{d}. Power of a, twice the Log, or two of the equal Spaces bb; the Expo^{t}. of the Cube of a, thrice the Log: of a &c. and make $bc.bc.bc.$ &c. as the Sq: Cub: biq: &c. Powers of a, to expound truly the flowing Areas a^{v}. Now, the whole Success of my Reasoning hitherto in Fluxions depending upon a Comparison betwixt the variably & uniformly described Spaces in equal or the same Portions of Time, let us also try here what can be done that way. . . . Suppose, therefore, these successive Additions of Log: a, representing continued Impulses to accelerate a Body c in Directions $\perp$ bD whilst it likewise is uniformly carry'd // bD; I say, suppose those to cease from B and the Body to go on with the Velocity BC there acquired.

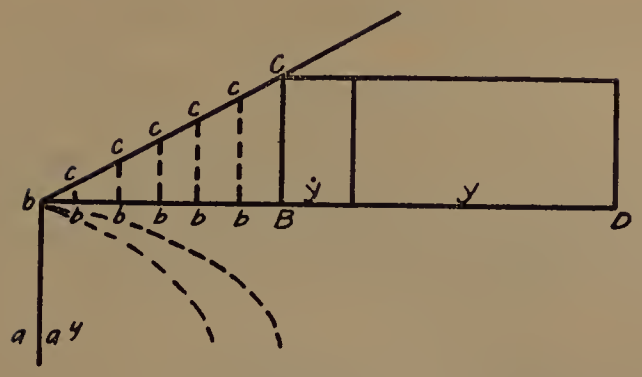


Fig. 16

Then 'tis evident the bc or a^{v} immediately preceding $x1:a$ produces the uniform Velocity BC or a^{v}, which $\times^{d}$ by $BD = Bb = y$, is the uniform Space producible in the same Time. Therefore $a^{v}\ 1:a \times \dot{y}$ is the Flux^{n}. of that Space, viz the Fluxion of a^{v}.— I have very great Hopes of this Method, but I shall not however build too confidently on it 'till you encourage me.

"There are a great many pretty Cor. to your Proj: Prob: [projectile problem]. . . . "

Simpson found Blake's reasoning difficult to follow, and, as is evidenced by his written answer, felt it necessary to tell him so:

"Honoured Sir,

"I have considered both your last Letters with an Attention suitable to ye Intricacy of ye Subject, but cannot say I clearly understand some things therein laid down, nor how far they conduce to ye End you are in pursuit of: For as to ye Infinite Succession of Curves, it may serve well enough to shew that a^{v}, or even x^{v}, may be expressed by a variable Area; but then I don't see how ye Fluxion of that Area can be obtained, from any property of y^{e} Curve, but only by supposing y^{e} Fluxion of a^{v} to be first known, which is y^{e} very thing to be determined. Your second Method seems to contain something of Importance, but here y^{e} great Difficulty is how a^{v} *multiply'd by* $1a$ *produces* y^{e} *uniform Velocity* BC, all y^{e} rest I understand perfectly well and did this appear in y^{e} same Light I would look upon y^{e} Invention as compleated. Indeed I don't perceive what use y^{e} variable

Area a^{y} is of in this Consideration or that there is any Connection between ye two Methods. . . . "[24]

There is a decided break in the correspondence between Blake and Simpson during the months of January and February, 1742, but a letter from Blake to Simpson dated March 28, 1742, would indicate that Blake's reasoning was leading him into new pitfalls during this interval. An idea of the nature of his difficulty becomes apparent in reading the following letter:

"Dear Sir,

"I stand corrected like the Blunderbuss as I am; for there can be no Doubt the Velocity of *F* is uniformly accelerated when the Impulses vary by a common Difference, whatever ye Progression be, 1.2.3.4.5: 1.3.5.7.9: 1.5.9.13. they are all x^{1} & the Intensity of the Impulse is only denoted by the Increase of ye common Difference.

"Nor is it less obvious y^{t}. the Orders are limited when any other direct whole Pow^{r}. of x is put for the Impulse, as in 1.4.9.16:x^{2} expounds it, where 1 + 4 + 9 &c. make *EF*, 3.5.7 *GH*, 2.2.2 &c. *IK*, and *LM* is constant. But still I don't think it so clear from the Nature of Motion y^{t} the Orders become infinite when the Impulses are inversely as any Power of x; & surely some such Illustration must be necessary, since I may be suppos'd to labour under no more Difficulties than many of my Readers; and indeed it is not obvious (for Instance) how x^{-1} sh^{d}. occasion an infinite Succession of Fluxions, when x^{-1} is limited to 3. I say this does not appear obvious from the Nature of Motion, tho' I agree in taking the Flux^{s}. of x^{-1} or $\frac{1}{x}$ it is analytically proved by the continual Increase of the Index of x,— I know no Way of solving the Difficulty at present but this.

"The Orders of Flux^{s}. are limited so long as n (the Index of x) is a positive whole N^{r}. & finite, but n being infinite they also become infinite, & the negative Powers of x are more than infinite. But this; if even right, is not fit to satisfye Beginners I'm sensible; so I shall endeavour at some other Way, & in the mean Time w^{d}. be glad to hear for my own Satisfaction whether

[24] Letter written January 2, 1742, from London.

or no $x^{-1}.x^{-2}$.&c. have got their negation by passing thro' Infinity & thence produce an infinite Succession of Fluxions as x^n does when n is infinite.

"And to this you will be pleas'd to add your Opinion, what Part of Physicks will be most proper for Examples to the Essay, . . . If I did not know you to be a man of the strictest Veracity, I c^{d}. believe Dr. Bevis was not alive. I really thought he got into this Comet; however, as I'm Sure he is pretty well acquainted with it, tell him I beg he will send me his Observations, or some Account of them. I remain,

"Dear Sir,

"Your most Obliged Humble Servt.

"Francis Blake"

"West Herrington, March 28, 1742.

"P. S. When I write you any Thing absurd, I'm sure you do or will do it the Justice to commit it to the Flames. . . . Now for a good Problem.—"

In his next communication with Simpson, written April 2, 1742, Blake attempted to answer his own difficulties with regard to a proper explanation of the cases in which a progression of fluxions is either infinite or limited:

"Dear Sir,

"Not many Hours after I sent you my last Letter, I began to think 'twould not be difficult to explain universally when a Progression of Fluxions is infinite & when limited. Now, I say ye Progression is limited when the Qtys, expounding the Forces are to each other as N^{r}. to N^{r}. and on the contrary infinite when they can only be express'd in infinite Series. For *EF* is the Sum of the Forces, & the succeeding Flux^{s}. arise from the Differences in these Forces: but if the Forces themselves be incommensurable, a Convergency of Differences, &, consequently of Fluxions in Infinitum will ensue. Thus x^1 shews the Forces to be as N^{r}. to N^{r}. 1.2.3.4. &c. and the Differences are 1.0. whence there are no 4th Fluxions. In x^2 the Differences are 3.5.7. &c. 2 2 2 .000 where no 5th Fluxions happen.

"On the other Hand $x^{\frac{1}{2}}$ gives an infinite Progression, for the Sq: Roots of the Natural Numbers 1.2.3.4. &c. are only had in infinite Series, & the Differences of Differences &c. converge in-

finitely. So again $\frac{1}{x}$ or $\frac{1}{x^2}$ &c. are Irrational, wherefore Fluxions are in all such Cases endless.

"If these things be right I shall think myself pretty well master of my Subject at last. *But you'll be amaz'd to hear y^t I have thought of introducing the Essay, sh^d. it be reprinted, with a Defence of Sr. Isaac Newton's Method. Our Disputants seem to me to make sad Work of it.* The whole Debate is (if I understand them) how

$$\underline{x^m \qquad A \qquad B}$$

$mx^{m-1}\,\dot{x}$ sh^d. expound the uniform Space with the Velocity at A. And Pray, what can be plainer? S^r. Isaac says AB is too great because of the Accelerations betwixt A & B; and therefore to exclude these Accelerations he supposes AB in its' ultimate State or (which is the same) B coinciding with A; whence $mx^{m-1}\,\dot{x}$ is producible (not produced) by the very Velocity at A & none other. Some of the Champions certainly imagine y^t S^r. Isaac says $mx^{m-1}\,\dot{x}$ remains of AB, tho' AB is an evanescent Quantity. . . . "

Meanwhile Simpson labored under difficulties in trying to reconcile some of Blake's ideas with his definition of Fluxions. Commenting upon Maclaurin's work, Simpson remarked:

"As to what Mac Laurin has done in y^e Business of Exponentials I don't think it will at all interfere with your Design; for tho' he has gone a great way about ye Bush, he seems not to have hit ye Mark. . . . "

Blake also mentioned Maclaurin's work, when writing to Simpson of an application he had evolved to Simpson's "projectile problem." Writing from Twisel, Blake says:

"Dear Sir,

"Your last Favour was forwarded from Herrington to me at Twisel; and, if this reaches you before you answer my Letter of the 20th of June be pleased to direct to Herrington again, for I propose to leave this Place today.—

"I see plainly a Part of the centripetal Force is destroy'd by the centrifugal Force, which arises from a Projection in the Tangent, and, no Doubt, the Remainder is the whole centripetal Force

employ'd to keep the Body in a Curve. And, however difficult the Method may be, I am quite of Opinion the most natural Way of estimating that Force is to deduct the centrifugal from the whole rectilineal centripetal Force; for the Difference is evidently the centripetal Force required. . . .

"I conceive a Force towards and another from ye Center to act every Moment; and as both these are 2^d^. Fluxions, the 2^d^. Fluxion of the one,—the 2^d^. Fluxion of the other, gives it in algebraic Terms. Again, the Ratio of these Forces will, I think, determine the Apsides, & shew when there can be but one Apse & when the Body must describe a Spiral; which I find are Mc Laurin's Speculations but he seems to have handled them somewhat clumsily.— . . .

"Give my Service to Dr. Bevis & tell him I hope he will never break his Leg again, tho' he sh^d^. never learn Fluxions. But as I believe he can scarce live with^t^. straining his 3^d^. leg, I w^d^. recommend that Opportunity, since Water—Grace and Physick are Friends to Study.—I am,

"Dear Sir,

"Your Assured Friend, and

"Servant

"Francis Blake."

"Twisel, July 14, 1742."

"P. S. You may see how fond I am of your Project: Prob: by endeavouring to apply it on all Occasions. The above Process is only that with the Ideas of the Symbols vary'd."

Maclaurin's *Treatise of Fluxions* was published in 1742, and from the date of its publication Blake suffered from anxiety as to whether the similarity of the work to his own was very apparent. The lemma, "The Fluxion of the Area *ABC*, whether triangular or curvilineal, is the rectangle $\dot{x}y$," included by Blake in his essay, is one which is elaborated in detail by Maclaurin in his work.[25] On this point Simpson sought to reassure him:

"Honoured Sir,

"I cannot think you have the least Cause to be under any concern at what y^e^ World may suggest with regard to your Essay, because it was not only published long enough before Mr.

[25] See supra, pp. 61, 62 and pp. 69, 70.

Mac Laurin's Book appeared,[26] but because what you have done therein is vastly more clear and satisfactory than any Thing to be met with in that Author; and this is not my bare Opinion but ye Opinion of all that I have heard speak about ye two performances.

"And if I might freely declare my Mind I should rather think he made that Alteration in his plan in Consequence of having seen your Essay than that you had borrowed any Thing from him. But be this as it will, I am confident that a much plainer Book may be wrote on ye Subject than his, and that an easy Explication of ye Theory illustrated by a Sufficient Number of well chosen Examples, will be of much greater use to a Beginner than both those large Volumes—

"Your Method of considering ye ascent of Bodies I approve of so far as I understand it, but in ye algebra there are some slips (if I mistake not) which render ye process a little perplexing. . . .

"Since ye Arrival of your last Letter I have been trying to investigate ye Centripetal Forces of Bodies moving in Conic Sections according to ye Method therein described but have found ye Matter so perplexing that I have been obliged to desist from that Enquiry without producing any Thing worthy your Perusal; tho' it was in hopes of communicating something on this Head that was ye sole Cause you have not heard sooner,

from, Hon^d^. Sir,

"your most obliged and

"most obedient Serv't.

"T. Simpson."

"London, August 7, 1742."

"P.S. I cannot omit acquainting you that Mr. Jones author of y^e^ *Synopsis*,[27] is so well pleased with your performance that he was very desirous to know who was ye Author of it, and seem'd to express a Satisfaction when ye Doctor told him."

Blake's next communication with Simpson, however, does not express the same confidence which Simpson displayed in his. In the letter, written September 21, 1742, Blake says,

[26] See supra, pp. 69, 89.

[27] *Synopsis Palmariorum Matheseos*, London, 1706; See Smith, *History of Mathematics*, II, 312.

"Dear Sir,

"You are very good to stir me up to the Pursuit of Mathematics; but I can assure you that it is my greatest Pleasure, & consequently I am far from being dispirited or weary therein. What I said of Fluxions was because of the Confinement to a particular study preventing a more general Survey of Things: tho' as all may be easy, after so abtruse a Doctrine, you are right to advise me to proceed in it; and I shall accordingly try what I can produce against the Time that your Treatise will again be printed. . . . [28] *But you don't tell me whether or no McLaurin has hit upon the Method of considering Exponentials*, proposed in my Letter dated Jany. 29, 1741. Perhaps indeed, *as I have the Book you think it an odd Question; but I vow he is so tedious & confused y^t I can't well determine what he would be at: yet I believe he has miss'd it.* He seems to come nearest at Articles 176, 177. Tell me in your next. . . . "

This work on *Fluxions*[29] by Maclaurin, which received so much adverse criticism from both Blake and Simpson, was one of which Playfair[30] (c. 1795) wrote, "At the moment when we now write, the treatises of Maclaurin and Simpson are the best which we have on Fluxionary Calculus." Playfair also admitted that, excellent as he thought these treatises to be, they were still inadequate to the state of science in his day. It is at present, however, generally considered the ablest and most rigorous text produced during the eighteenth century.[31]

During the next two years the mathematical topics discussed by Blake in his letters were very similar to those in the preceding years. The fluxionary calculus seems to have been his main interest.

He attempted to produce something on trajectories, and an improved method of finding the "Apside angle," but finally, because Simpson had already treated it adequately, he dismissed

[28] Evidently Simpson was at this date contemplating the 1750 edition of the *Doctrine of Fluxions*.

[29] *A Treatise of Fluxions in Two Books*, by Colin Maclaurin, A.M., Professor of Mathematics in the University of Edinburgh, and Fellow of the Royal Society, Edinburgh, MDCCXLII.

[30] John Playfair, *Collected Works*, vol. IV, p. 323.

[31] See Cajori, *History of Fluxions*, p. 181.

the topic, with the statement, "It was ridiculous for me to attempt any Thing in Trajectories, when you have given so universal and elegant a Solution in your Essays; but indeed I did not consider it enough when I writ to you." From this he passed on to a consideration of infinite series. In his study of the subject of infinite series, however, Blake fell into difficulties, and Simpson was obliged to explain a troublesome part which, by his own statement, he had previously been asked by Mr. Muller to make clear.

Simpson also included some interesting side-lights on the mathematical matters of his day. For instance, he wrote:

"The person to whom many give the Honour of your *Essay* is one *Finch*, an ingenious young Fellow, from Norwich, who went to Sea about 3 years ago, and give it out before he went that he wrote a small piece upon Fluxions, and sold ye copy to Mr. Innys; but others ascribe it to Mr. Smith, not the professor, but another gentleman of that name.

"As to the privilege of being admitted a Fellow of ye Royal Society, it is not so easily obtained now; as it was some time ago; for, then such numbers were entered without any sort of Merit, that ye Society (as I am informed) have found it necessary to come to a Resolution not to receive any person as a Member who has not first distinguished himself by something Curious. But this is no Barr to you since ye Book you have wrote (or something you may publish in ye Transactions hereafter) will be sufficient recommendation for you; would your modesty but permit you to own it.

"I cannot help informing you that Mr. De Moivre[32] has attack'd me in my Book of Annuities, and that in a most ill natur'd[33] Manner. . . . "[34]

The following May [May 7, 1743] Simpson referred again to De Moivre:

"Honoured Sir,

"I was under some concern I did not hear from you, but hop'd ye Reason might be only a Hurry of Business, and that I should have ye Pleasure of seeing you in Town ye sooner.

[32] *Abraham de Moivre.* See Smith, *History of Mathematics*, I, 450.

[33] This was an attack on his book, *The Doctrine of Annuities and Reversions*, London, 1742. See below, Chapter V, pp. 95–99.

[34] Letter written by Simpson to Blake dated March 24, 1743.

"I cannot forbear declaring how much I think myself Honoured and obliged by your generous Assurance of Friendship and ye Respect and Freedom with which you express your self.

"The Book of Fluxions, which you desire a Character of, is, in my Opinion a very ordinary immethodical Performance. The Principles on which it is grounded are extremely dark and unsatisfactory & the Manner of Expression very inaccurate and obscure: There is little, or nothing in it, new, and, if anything can recommend it, it is a large Table of Fluents and an Application of Fluxions to a great variety of physical Problems. I have nothing new to inform you of only that I have just wrote a small pamphlet in answer to Mr. De Moivre which will be published next week.

"I am

"Hond. Sir &."

"London May 7, 1743."

"P. S. The Principles are nearly ye same with those of Ditton[35] but rather more obscure; and he has less upon 2^d^. Fluxions than I have."

Preparation of a pamphlet in answer to De Moivre was characteristic of Simpson, even though De Moivre's reputation in the particular field of the doctrine of annuities and reversions was far greater than his own.

So far as can be determined Blake never republished his *Essay* with the improvements he contemplated, although such a publication would have had value.

The correspondence between Blake and Simpson was of great service to both. Not only did two very good treatises result from it, which were well received by the public,[36] but both of these men were able to clarify their ideas in a way that was not

[35] He refers here to Humphrey Ditton, who in his work most closely followed Newton. Ditton brought out in 1706 in London a work popularly referred to as *An Institution of Fluxions*; see Smith, *History of Mathematics*, I, p. 456.

[36] Simpson's *Essays* were almost sold out by the end of 1740, for in a letter to one of his booksellers, Simpson says,

"I return you many thanks for your advice about advertising in ye Newcastle papers, but as I have only a small number left I hope to dispose of them without being put to any additional expence." This letter was written, December 30, 1740.

possible under less favorable circumstances. Although the relationship between them was supposed to be that of pupil and teacher, both benefited and grew intellectually through the correspondence. Nothing can better show Simpson's personal advancement in mathematical science than his letters to Blake throughout this period. In addition, their correspondence represents the type of mathematical thought and conversation of that time.

CHAPTER V

Publication of Treatises

THE DOCTRINE OF ANNUITIES AND REVERSIONS

Simpson had already written a treatise on the *Nature and Laws of Chance*, a subject on which De Moivre was the acknowledged authority of his day. He now turned his attention to another subject made famous by that author. In 1742 Simpson published in London a treatise entitled *The Doctrine of Annuities and Reversions*.[1]

De Moivre criticized the work in the preface to the second edition of his *Annuities on Lives*, and, in spite of the fact that Simpson had complimented De Moivre highly, he included in his criticism some harsh reflections upon Simpson. Consequently Simpson published in the following year, *An Appendix, containing some Remarks on a Late Book on the same Subject* (By Mr. Abr. De Moivre, F.R.S.) *with Answers to some Personal and Malignant Misrepresentations in the Preface thereof*. In the close of this very effective answer, Simpson concluded, "Lastly, I appeal to all mankind, whether, in his treatment of me, he has not discover'd an air of self-sufficiency, ill-nature, and inveteracy, unbecoming a gentleman." This seems to have ended the dispute because De Moivre removed his "remarks" from the preface to the third edition of his *Annuities* which was printed in 1750. Although Simpson did not alter his treatise in the light of De Moivre's criticisms, he did publish a short essay in 1752 with his *Select Exercises*, which he intended as a supplement to his work on *Annuities*. It was not, however, until 1797 that this essay was printed with the *Doctrine of Annuities*. This supplementary essay was better known during the eighteenth century than was the original treatise, for in it Simpson had published a valuation for single and joint lives calculated according to the London bills

[1] *The Doctrine of Annuities and Reversions; deduced from general and evident Principles: with useful Tables, shewing the Values of Single and Joint Lives, &c.*, 8vo.

of mortality. Later writers and pioneers in developing the subject of insurance used these tables freely.

Influence of the Work on Later Writers. An examination of the efforts of later writers on *Annuities* reveals Simpson's influence. Charles Brand, one of the group of later writers,[2] made use of the tables published in Simpson's work on the *Doctrine of Annuities*, but failed to take advantage of the more correct rules for the valuation of annuities as found in the supplementary essay. A pamphlet published in 1777 by Samuel Clark offers objections to the corrections applied to the solutions to problems given by De Moivre and Simpson, as explained in a treatise by Dr. Richard Price in 1775. Price[3] had employed Dr. Halley's tables instead of Simpson's tables calculated from the London bills of mortality, in solving the problems in Simpson's treatise, and consequently arrived at different results. Backhouse, another of the later writers, borrowed chiefly from Simpson for his definitions and problems relating to the doctrine of chances and annuities.[4] Maseres,[5] while attempting to make the subject of annuities intelligible to a person unacquainted with the doctrine of chance, has included the best material he could find for his purpose among the treatises of Halley, De Moivre, Simpson, and Price.

General Interest in the Doctrine of Annuities and Reversions. When one realizes the life of the Englishman of the eighteenth century, it is small wonder that Thomas Simpson felt called upon to write an elementary treatise on the doctrine of annuities and reversions. As Dodson summarized the situation, "The present possessors of entailed estates, are in common law justly called

[2] *A Treatise on Assurances and Annuities on Lives; with several Objections against Dr. Price's Observations on the Amicable Society and Others, etc.*, London, 1775.

[3] "Treatise on Reversionary Payments," *Philosophical Transactions*, 1775, v. lxv, p. 424.

[4] *A Dissertation on the Value of Life Annuities deduced from general Principles, clearly demonstrated, and particularly applied to the schemes of the Laudable and Amicable Societies of Annuitants for the Benefit of Age; with Tables adapted to the several rates. . . .* By W. Backhouse, London, 1778.

[5] *The Principles of the Doctrine of Life-Annuities . . .*, By Francis Maseres, London, 1783. For an account of this work see, *The Monthly Review*, 1784, v. 71, p. 358.

tenants for life; marriage settlements generally convey the reversion of a considerable part of the bridegroom's estate, to the bride, for her natural life after his decease; to which two things all freehold estates in these kingdoms are liable; and if to these be added, the great number of copyholds determinable on lives; and the estates possessed by ecclesiastical persons of all degrees, we shall find, that the values of the possessions and reversions, of much the greatest part of the real estates, in these kingdoms, will one way or other, depend on the value of lives.

"Likewise the incomes annexed to all places, civil and military, all pensions, and most charitable donations, are annuities for life; the interests or dividends of many personalities in the stocks have been, by wills of their possessors rendered of the same kind; besides which there are some annuities on lives which have been granted by the government, and have parliamentary security for their payment; and others have been granted by parishes and other communities, in consequences of acts of parliament made for that purpose."[6]

Correspondence with Regard to the Doctrine of Annuities and Reversions. The practical development of the doctrine of annuities during the eighteenth century was the origin of many of our modern theories of insurance. The popularity of the subject is seen in Thomas Simpson's extensive correspondence in answer to the many queries he received. Most of the questions he was called upon to answer are of a practical nature, as the following letters will show:

"Sir,

"As ye value of ye Fee-Simple (at ye Rate of 4 Cent), is only 25 Years purchase the value of Ten Thousand Lives cannot amount to 25 Years Purchase: But ye Value of one Single (pick'd) Life is worth 15 years purchase. Hence it appears, that, if ye Numbers of Lives (or Trustees) was to be infinite, ye Fine could not be increased in so great a proportion as that of 25 to 15, and therefore can never amount to ye sum £457. In hopes that what is here offered will reconcile ye Matter to your Judgment, . . . "

A draft of a letter written by Simpson to one of his corre-

[6] *The Mathematical Repository*, by James Dodson, London, 1753, v. 2, preface.

spondents, October 29, 1751, from Woolwich, is even more enlightening as to the type of question Simpson was often asked to answer:

"Honourable Sir,

"Below you have Answers to all your Questions; being ye Result of Calculations founded on a great Number of Observations on ye Bills of Mortality of different places.

"Question 1. 'How many Year's purchase must be deducted from ye price of an Estate in Fee Simple, in consideration of ye joint Lives of a Woman 57 Years old, and of a Woman 37 Years old; both of them but moderately healthy?'

"Ans. This Question, I take to be ye same as to find ye present value of an Estate to continue as long as either of ye two proposed Lives is in Being. And ye Answer to it comes out 14 1/8 Years purchase at 4 p. Cent., 15 Years purchase at 3 1/2 p. Cent., and 15 1/8 Years purchase at 3 p. Cent.

"Question 2. 'How many Years purchase must be deducted from ye price of an Estate in Fee Simple, in Consideration of ye Single Life of ye abovesaid person 37 Years old?'

"Answer. 12 Years purchase at 4 p. Cent., 12 4/5 at 3 1/2 p. C., and 13 3/4 at 3 p. C.

"Question 3. 'How many Years purchase must be deducted from ye price of an Estate in Fee Simple, in Consideration of ye single Life of ye abovesaid person of 57 Years old?'

"Answer. 8 1/2 Years purchase at 4 p. C., 9 at 3 1/2, and 9 5/8 at 3 p. Cent.

"Quest^n^. 4. 'What abatement must be made from ye price of any Estate in Consideration of a Debt of 18135£ whereof 2000£ pays an Interest of 4 p. Cent. and the remaining 16135£ only 3 1/2 p. C. All which is to be paid by the present possessor (i.e. the Woman of 37 Years old) and after her Decease ye purchaser of ye Reversion must take that Incumbrance, principal and Interest, upon himself?'

Ans^r^. 9250£ at 4 p. C., 9760£ at 3 1/2 p. C. and 10480£ at 3 p. C.

"You will see, Sir, that in these Solutions I have not taken upon me to assign ye particular rate of Interest that ought to be allow'd in ye Purchase under consideration: Nobody is a better Judge in

these Matters than yourself; for which reason I have been at ye Trouble to compute ye Values according to different Rates, in order that you may fix upon which you please. . . . "

RELATIONS WITH JOHN RANDALL

Preparation of a Textbook. Simpson had already attempted many things in writing and editing, but in 1743, we find him supervising the publication of a treatise by John Randall,[7] an acquaintance. Randall was the head of an academy at Heath near Wakefield, where Simpson's friend John Turner taught mathematics. Later, it is reported, he took six pupils to "reside with him at York." Throughout the year 1743 and well toward the close of 1744 Simpson labored with Randall in correspondence concerning printer's delay, expense of printing, corrections for the press, and statements of progress. Finally August 25, 1744, Simpson stated that the book had reached completion and that he regarded it as "a judicious collection of the various subjects therein treated of, and as a work worthy of encouragement for its real use and great comprehensiveness." From the remarks passed between the two men one would surmise that the book had a geographical part, and that the eighth chapter was mathematical. These conclusions agree with a work written by Randall, to be found in the British Museum, which was published in

[7] De Morgan in his *Arithmetical Books*, p. 74 in speaking of one of Randall's later publications, remarks,

"Mr. Randall was a quaint man, but his book [*An Introduction to so much of the Arts and Sciences, More immediately concerned in an Excellent Education for Trade in its Lower Scenes and more genteel Professions, &c.*, London, 1765] is well done. It contains arithmetic, mensuration, and geography; and ends with a dialogue between the heavenly bodies, upon their mutual arrangements, in which the earth insists upon being allowed to stand still, and quotes Scripture like an anti-Copernican, but is brought to reason by the arguments of others. This is almost the only writer I have met with who has given the student a few hints upon the habits of computation. Thus he will not let him say, three and four are seven, seven and five are twelve, &c.; but only three, seven, twelve, &c. For, he says, (the example being the addition of some rents) 'As you have this pretty Income, you must talk like a Gentleman to your Figures'."

It is thought that Randall obtained a B.A. at Christ's College, Cambridge in 1718, and the M.A. in 1727, (*Dictionary of National Biography*).

London in 1744 under the title of *A System of Geography.* Simpson appears to have given material aid to Randall in writing the eighth chapter concerning mathematics. In a letter written July 22, 1745 to Simpson, Randall apologized for neglecting to pay money due for his services, and thanked him in the following manner:

"I again heartily thank you for all your Care & Anxiety in the affair & shall be very glad to serve you in anything that may be in my power. The book has been very expensive; & the worst of it is, that there will be no gain, when all are sold off: so that I have my labour for my pains at present.

"You will give me pleasure I assure you in the continuation of your acquaintance. And in order thereto I invite you to exchange a letter with me at least, once a month. And I much wish, my converse with you thus, may reward you for your expence 4s. p.an. You gave me some hopes some time ago that you had thoughts of visiting your native country. If you continue in the same mind, I will do my utmost to meet you there.

"I have been broiling my brains again over your *Treatise of Fluxions;* & the more I study the more I admire you. You'll say I am mightily fond of writing when I tell you I intend to be in print[8] again very shortly. I have not time now, or I intended to communicate to you a plan for raising water by the Gravitation of the Air. . . . "

Correspondence about Teachers. The postscript to Randall's letter illustrates an interesting custom of the time. In it, he says, "Please Sir, to let me have a Line by the Time mentioned & shall thank you if you can help me to a complete Writing Master, a sober man and single. If you think he will deserve it I will give him 30 guineas & ann. & his board. It is not a boarding school [obliterated] for day scholars; & therefore he will have theless to do."

[8] The only treatises written by Randall to be found in the British Museum, aside from the works already mentioned, are, *An Account of the Academy at Heath Near Wakefield,* London, 1750; *A Course of Lectures in the Most Easy . . . and Entertaining Parts of Geography, Astronomy, Chronology, and Pneumatics, etc.,* London, 1750; *An Introduction to So Much of the Arts and Sciences, More Immediately Concerned in An Excellent Education for Trade, etc.,* London, 1765. I have been unable to locate any treatise written by Randall and published in 1745.

Many teachers were chosen to fill school vacancies through this method of appealing to a man of ability in the teaching profession to recommend either one of his students or some one in his acquaintance. Randall had great confidence in Simpson's judgment for in an earlier letter he had made the following similar request:

"Sir,

"I am very sorry I had not the pleasure of seeing you when I I was in London—I suppose things were so circumstanced on each side that it could not be—About Christmas last I desired two of the Teachers here who were then in London to visit on you in my Name, which I suppose they did but you were from Home. One was a clergyman, & the other a famous Writing master & Lover of mathematics, tho' he does not teach them here.

"I now trouble you Sir, upon account of Mr. John Turner's Death; he had been Teacher of Mathematics here some time, but died last Week. I shall think myself oblig'd to you if you can recommend me to one who is in every way qualified to profess the mathematics in this Academy, & who will be an Honour to it.—

"*I have many Youth grown up to Man's estate, who are sent here to finish their Education instead of going to the Universities; & this is the reason that I am oblig'd to get the best Teachers I can procure* ['lay my hands on,' crossed out], *for the Money.*

"The Order or Rank was the

Rev^{d}. Mr. Kipyeat; Chief Assistant (Fellow of a college).
Rev^{d}. Mr. Dodgson, Chief Classical Master.
Rev^{d}. Mr. Hyde, second Classical Master.
Rev^{d}. Mr. Norris, third Classical Master.

"Mr. Turner, Professor of the Mathematics.
"Mr. Gargrave, Chief Writing Master.
"Mr. Greaves, Music and Dancing Master; Man of Fortune.
"Mr. DeBoissy, Master of Modern Languages.
"Mr. Picard, Assistant to the above in the French, & Fencing Master.
"Mr. I. Row; Assistant in the Commercial School.
"Mr. Stancer; Assistant in the Lowest Latin School.
"Mr. G. Row, Assistant in the Commercial School.
"Mr. Crofts, D^{e}.

The Reason I give you this list is, that you may Judge what rank a Teacher of Mathematicks will have in this Society, & he may make his Life very happy here in this pleasant retirement. I would have him if possible a single man; & then I shall find him Lodging and Board; & as to his salary it can not now be named for I would give any price for a proper person; & when such is found, we can agree upon all particulars—I have paid the Postage to London—I shall take it as a favour if you will let me hear from you as soon as possible; otherwise I shall be pretty much incommoded with applications, which is always the Case, when there is a Vacancy here—, I am, Sir, . . . "

In answer to this request Simpson sent the following letter:

"Sir,

"Conformable to your Desire I take this Opportunity to recommend to you a Person, whom I look upon as properly qualified to teach ye Mathematicks in your Academy; being well skill'd not only in ye more common, but in ye most sublime parts of those sciences.

"He is about 30 years of Age; and tho' not a single man, has no Family, besides his Wife—I need not mention to you that he is a person of strict integrity, without which qualification I would not have named him; notwithstanding his other Merits; which are indeed very considerable; and amongst which may be reckoned a large share of Diligence and a remarkable Sobriety.

"His mathematical Character is well known here, about London, and even in ye Country, by Several ingenious Things he hath printed in a Work now publishing, in Numbers, called, *The Mathematician*; of which performance he is the chief Conductor. If any part of ye said Work has reached your Hands you will be able to form an idea of ye abilities of Mr. John Turner of London, ye Gentleman I am now writing of (In which performance ye place of abode is put down with his Name to distinguish from ye ingenious Mr. Jno. Turner of York your late Professor.

"As to ye Business of natural Philosophy (which you mention in particular) he is very well acquainted therewith, not only in ye Experimental but in ye Demonstrative part. He has not indeed had ye Opportunity of making many experiments himself: which you know cannot be done without proper (and expensive) Ap-

paratus; but as he knows ye Subject, so well I doubt not but he will be able to go through what is required with great Facility; especially as he is naturally very apt at mechanical devices.

"To conclude—he tells me he is willing to serve you upon proper terms; and therefore desires to know what you are inclinable to allow; For as he is already in a handsome employment (being at this time chief clerk to a considerable merchant) he thinks it would be imprudent to throw himself out of Business before he partly knows what he has to depend upon.

"I am Sir, with much esteem,

"your most humble servant, T. Simpson."

"John Turner of London" as Simpson styles him, was one of his private pupils during his career at London. The fact that Simpson had reached the stage of recommending people to fill vacancies is an indication of his own advance. His correspondence with Randall shows that he was at this time regarded as an authority on matters of publication and matters concerning the preparation of teachers.

The magazine entitled *The Mathematician* here mentioned, had as its chief contributors John Turner of London, and Samuel Clarke, both of whom were Simpson's students. Most of the problem material which is printed here can be found in Simpson's publications, and a great number of the problems used are taken from his *Treatise of Fluxions*. For this reason it was thought that Simpson had some interest in the publication,[9] or was its editor.

PUBLICATION OF MATHEMATICAL DISSERTATIONS

Simpson published a treatise composed of a number of essays in 1743, under the title of *Mathematical Dissertations on a Variety of Physical and Analytical Subjects*, in 4 to. He dedicated the work to Martin Folkes, Esquire, President of the Royal Society.

The publication was well received, a circumstance which is easily explained by its contents. Among other things, Simpson includes a "demonstration of the true figure which the earth, or any planet, must acquire from its rotation about an axis; also,

[9] *The Mathematician, containing a dissertation on the rise, progress and improvement of Geometry*, published by a Society of Gentlemen, no. 1–2, London, 1745; no. 1–8, London, 1750.

a general investigation of the attraction at the surfaces of bodies nearly spherical." Simpson has also discussed a "determination of the meridional parts, and of the lengths of several degrees of the meridian, according to the true figure of the earth."

Discussion of the figure of the earth was encouraged at this time by the prize offered in 1740 by the *Academy of Sciences* at Paris for the best investigation of this topic. The prize was shared by Maclaurin, Euler, and Daniel Bernoulli. Maclaurin's work was especially commendable for the new propositions it contained. In the first two papers of the *Dissertations* Simpson presents a discussion very similar to Maclaurin's work. In an account of the determination of the figure which a planet or a homogeneous fluid must acquire from its rotation about an axis, Simpson speaks of this resemblance:

" . . . the true Figure, under such Rotation, is not only universally demonstrated, but the particular species thereof, according to any assigned Time of Revolution; in which it is proved that the Gravitation in any Point in the Surface, is accurately as a perpendicular to the Surface at that point, produced from thence to the Axis of Revolution; and that it is impossible for the Parts of the Fluid ever to come to an Equilibrium among themselves, when the Motion about the axis is so great as to Exceed a certain assignable Quantity; with several other particulars not touched upon by any. I must own that, *since my first drawing up of this Paper*, the World has been obliged with *something very curious on this Head, by that celebrated Mathematician Mr. MacLaurin, in which many of the same things are demonstrated.* But what I here offer was read before the Royal Society (March or April 1741) and the greater Part of this Work printed off, many Months before the Publication of that Gentleman's Book; for which Reason I shall think myself secure from any Imputations of Plagiarism, especially as there is not the least likeness between our two Methods."

Yet in spite of the priority to which Simpson lays claim, Maclaurin was really the "creator of the attraction of ellipsoids," and Maclaurin's researches were independent of those of Simpson. Todhunter has made this contention,[10] and a letter

[10] Todhunter, Isaac, *A History of the Theories of Attraction and the Figure of the Earth*, 2 v., London, 1873, I, 176–188.

from Maclaurin to Stirling on December 6, 1740,[11] confirms it. At that time Maclaurin favored Stirling with an account of his method of treating the attraction of spheroids and the figure of the earth.[12] Yet while evidence shows that Simpson's work will not answer to the claim of priority to Maclaurin's, it is nevertheless an independent piece of research, and according to Todhunter, "very remarkable as we shall see by an analysis of its contents."

Discussion of the figure of the earth led to a study of the theory of tides. In the third paper of the *Dissertations*, Simpson has given an investigation of the heights of tides in the ocean.

In addition, Simpson has included in the *Dissertations* a discussion of a new theory of astronomical refractions, "with exact tables deduced from the same;" a new and exact method for approximating the roots of equations in numbers, which "quintuples the number of places at each operation;" a determination of the time of year when the days lengthen the fastest according to an "assigned eccentricity of the Earth's Orbit;" and a discussion of how much the descent of bodies is affected by the earth's rotation. Among the methods for the summation of series which Simpson has given, the rule known to modern textbooks as Simpson's rule occurs.[13] While known by Simpson's name, the parabolic rule was first given by Cavalieri. Finally, some new and useful improvements in the inverse method of fluxions are included in this collection of essays.

[11] *James Stirling, A Sketch of His Life and Writings Along with His Scientific Correspondence*, by Charles Tweedie, Oxford Clarendon Press, 1922, pp. 91, 92.

[12] He writes, "I begin with what I sent you two years ago, but the demonstration is somewhat different. . . . "

[13] *Mathematical Dissertations*, p. 109.

CHAPTER VI

Appointment to Woolwich and Close of London Career

APPOINTMENT TO WOOLWICH MILITARY ACADEMY

During the same year in which Simpson was aiding Randall in his publication, and in which the *Mathematical Dissertations* appeared in print, he received his appointment to Woolwich Military Academy. It has been said that his appointment was due to the fact that he dedicated the *Dissertations* to Martin Folkes. Hutton, however, says, "Through the interest and solicitations of the before mentioned William Jones, Esquire, he was, in 1743, appointed professor of mathematics, then vacant by the death of Mr. Derham, in the Royal Academy at Woolwich; his warrant bearing the date August 25th. . . . " While it may have been the result of the joint efforts of these two men that Simpson was appointed, much light on the efforts of Martin Folkes' in his behalf is obtained by a study of the Simpson-Folkes correspondence.

Correspondence with Martin Folkes Relative to Appointment. As early as September 1, 1741, Folkes wrote Simpson speaking of the fact that he was willing to do anything in his power to "intercede in his favour to the Duke of Montagu," whom he characterized as a very human man.

Simpson had at this time suffered an attack of ill health which had necessitated a journey to Bosworth to regain strength. After wishing him a complete recovery, Folkes added,—"as you are able I wont give it as my advice that You should not neglect whatever is in your Power to do in your part, which will always be the best argument your friends can use in your behalf in case of any application made, which I hope there is not to supercede you. . . "

The efforts of Folkes in Simpson's behalf culminated in 1743 when on August 25, Simpson received the following letter from him:

"Sir—

"It is with great satisfaction I can acquaint you that his Grace the Duke of Montagu has been pleased to appoint you second master of the Royal school of Artillery at Woolwich in the room of Mr. Derham deceased, and has signed a warraunt for you this morning accordingly.

"I give you joy on the occasion and am well assured i need not recommend your behaving with the greatest duty and gratitude to his Grace on every occasion, who has so readily bestowed this favour upon you. I look on myself very greatly obliged to My Lord Duke for his kind notice he has been pleased to take of my recommendation of you; and as I have given my word not only for your skill and abilitys but also for your diligent and exact attendance on the dutys of your part, I am persuaded you will have the greatest regard to what I have promised for you and that you will neglect no opportunity of serving the publick in this office to his Grace's satisfaction, who is in all respects the best of masters.

"I shall also take the liberty of recommending to you the cultivating with the greatest care the best understanding with the first master, whom it is very necessary you should act in concert with. You will also I doubt not take care to pay all proper respects to all your superior officers, and be punctuall in everything that concerns your behaviour to them. You will consult with Mr. Muller about your method and the proper things to be taught, which I suppose should after the first elements be rather the practical parts from the deeper Geometry unless where a genius may be found fitted for them; and this will also be easier to yourself and leave you leisure when not employed in duty, to pursue yourself those higher speculations, which will enable you to give yet more useful and valuable discoveries to the publick. When you have your warrant which I understand will be about Tuesday next, you will as soon as conveniently get in readiness to remove to your predecessors lodgings there, whither it will be expected you will go as soon as may be, and take upon yourself the instructing persons you have to take care of, according to the directions you may receive from the Duke or other proper officers under him. I truly sir wish you all health and happiness in your new situation, that you may have all success in your business,

and give both his Grace and all others that satisfaction I am well persuaded you will. I wish you would call upon me as soon as you can this afternoon when I will give you a line to the Duke's secretary whom it will be proper you should wait upon as soon as may be for instructions about taking out your warraunt from the Office of Ordnance.

"I am sir very truly,

"Your faithful humble servant, M. Ffolkes."

To have enlisted the friendship and protection of so great a man as Martin Folkes was extremely fortunate for Thomas Simpson. Folkes was a graduate of the University of Cambridge, and had the degree of Doctor of Laws conferred upon him in July 1746 by the University of Oxford. His whole life was devoted to the Royal Society of London. He held various offices from the time of his admission as a fellow in 1714 until his appointment as its president. He was a close friend to Sir Isaac Newton, so that in dedicating the xxxiv volume of the *Philosophical Transactions* to Folkes in 1727, Jurin, Secretary to the Royal Society, gave as his motive: "the same which induced the greatest man [Sir Isaac] that ever lived to single out Mr. Folkes to fill the chair, and to preside in the assemblies of that Society, when frequent returns of his indisposition would no longer permit him to attend them with his usual assiduity. This motive we all know was an uncommon love to and singular attainments in, those noble and manly sciences, to which the glory of Sir Isaac Newton, and the reputation of the Royal Society, is solely and entirely owing." Jurin concluded his dedication with the statement, "It is sufficient to say of him [Martin Folkes], that he was Sir Isaac Newton's Friend." Compared with earlier friendships, acquaintance with Martin Folkes indicates for Simpson a great change and more intellectual companionship.

Letters of Congratulation upon his Appointment. Pupils and friends took advantage of the opportunity to congratulate Simpson upon his appointment to Woolwich. F. Holliday of Hertford, who signs himself "Your pupil," and encloses some problem work which Simpson had given him to do, closes his letter with the statement: "I was in town a Week ago in order to have spent about four or five days with you but you was out of Town. I am very glad to hear of your promotion to Woolwich, which Mr.

Key told me of. I cou'd wish to be your Reader there amongst the young Gentlemen."[1]

Another letter which Simpson received at this time is from a pupil and friend of long standing, signed by E. Fenwich and W. Fenwich. It is one of the few which Simpson ever received mentioning his family relationships. Fenwich writes a sincere congratulatory message. Of Mrs. Simpson he says,

"My kind Love and Service to Mrs. Simpson and tell her I and my Wife wish her Such Joy as she may or can wish for and a Continuance of the Same all the Days of Both your Lives. Miss Polly we wish well. . . . "

I have been 2 Sunday's Meeting my Masters from Bath. I Labor like a Turk but all these times that I miss wch is thrice I enter'd as due to You which I will honourably pay but at present paying three pounds a month in to My Dear Friends hand will be compleated by ye 1 of April next & then oh how happy Shall I be my not meeting of You will be no Loss to You. I hope to See You next Sunday."[2]

During the summer of 1743 Blake spent some time in London. This we infer from a letter written to Simpson in August of that year, apologizing for leaving without bidding him farewell. On September 6, 1743, he writes a congratulatory letter upon Simpson's appointment to Woolwich:

"Dear Sir,

"I most heartily congratulate you upon your preferment. Mr. Folkes has shewed himself in this, as in other matters, to be a wise and good man. And I hope you'll excuse me that I offer

[1] Letter dated Hertford, September 18, 1743, to Mr. Simpson, Teacher of the Mathematics, Crown Court, Hog Lane, Moorfields.

This is probably the Francis Holliday who is responsible for several mathematical works. To him is due a translation of Stirling's *Methodus differentialis* into English "with the author's Approbation," London, 1749; a treatise entitled *Miscellanea curiosa Mathematica;* and finally *An Introduction to Fluxions, Designed for the Use, and Adapted to the Capacities of Beginners*, Nott's, London, 1777. The last work was published after he had become Reverend F. Holliday, Vicar of West Markham and Bothamsall. Cajori, *History of Fluxions*, p. 243, gives a review of his work on fluxions.

[2] Letter dated December 1, 1743, to Mr. Simpson, Theory Master at The Royal Academy, at Woolwich.

you here a small compliment to help you to take your clothes off the Tenter-Hooks and remove them to Woolwich. . . . "

He concludes with some mathematical material:

"In the method I propose to treat Gravity, I utterly exclude a succession of impulses & embrace the Idea of a continued force; whilst at the same time I endeavour to explain the phenomena mechanically; but the whole is in such a way as to be quite subservient to fluxions by tending to remove all obscurity from that doctrine since some there still are I doubt not, especially amongst beginners. As for the Doctor's attacks I must receive them as well as I can.[3] When I was with Mr. Folkes in town, he mentioned a peculiarity in the structure of a Honey Comb which gave rise he said to a pretty difficult problem. It relates to the bottoms of cells which are not one plane, but each is composed of three triangular planes forming a solid angle, from whence it is evident the hexagons are not regular, that is, not of equal sides. . . . It is required to change the sides of a regular hexagon so that preserving the length *AB* the 3 triangular planes *ADE. DFB. EQB* turn'd up, may unite in a solid angle and form a solid or vessel which shall be a maximum. . . . "

Simpson, meanwhile, is evidently busy attending to affairs relative to his removal to Woolwich Warren, the location of Woolwich Military Academy, concerning which he says:

"Mr. Folkes has been so good as to recommend me to ye place of second Master at ye Royal Military Academy at Woolwich (lately become vacant) which, thro' his Interests, I have obtained (in opposition to a great many competitors) and not knowing 'till now how the thing would be, nor how soon I might have occasion to move. I defer'd writing therefore till I cou'd inform you of my success and how to direct me. As I shall now have very little Company and a great deal of Time upon my Hands, your correspondence will, if possible, be more desirable than ever, and I am pleas'd that ye Opportunities which I shall have in my new Situation, in seeing and trying Experiments may, probably, help furnish out matter towards carrying on such a Conversation.

"I shall be extremely glad to hear that you have succeeded in your Attempt upon Gravity and Fluxions.—You already know

[3] He no doubt refers to Dr. Bevis jokingly.

my opinion of ye Matter—Indeed I am not positive, but ye Doctor threatens to demolish you at once. . . . "[4]

Thus a rather spectacular career of private teaching drew to a close, and the next year was to find Simpson assuming another rôle,—that of a teacher in an academy. His London career had been one of many friendships, many publications, extensive correspondence, and many contacts with the mathematical world of his day,—especially that of the lower and middle classes. His influence during this period can be judged by the careers of his private pupils, in addition to the influence of his publications.

CAREERS OF SIMPSON'S PRIVATE STUDENTS

Many of Simpson's private pupils became leading men in the field of mathematics through his recommendation. As has already been mentioned, Francis Blake wrote a usable treatise on fluxions. John Turner became a professor of mathematics at Randall's Academy at Heath, near Wakefield. Francis Holliday became an author of a variety of textbooks on mathematical subjects, as well as translator of one of the greatest productions of the century, Stirling's *Methodus differentialis.*

The future reveals notable careers of other of his students. Upon James Dodson's death, Mr. John Davis a student of Simpson's for several years, campaigned according to the custom of the century for the place vacated in the Royal Mathematical School in Christ's Hospital. The circular sent out in this connection read as follows:

"Sir,

"I humbly intreat the Favour of your Vote and interest to succeed *Mr. James Dodson,* deceas'd, as master of the Royal Mathematical School in *Christ's-hospital;* and should I be favour'd with your Approbation, shall ever testify my Gratitude by a faithful and diligent Discharge of the Trust reposed in

"Sir,

"*Your most obliged,*

"*And most obedient Servant,*

"JOHN DAVIS."

[4] Letter dated September 3, 1743.

"We do personally know Mr. John Davis to be a man of Sobriety and integrity; that he has been regularly educated in the study of the mathematics, and has acquired an extensive knowledge in most considerable Parts thereof; that he was a warranted School-Master in the Royal Navy during part of the last War, and is now established in the Business of teaching Navigation, and other collateral Branches; We therefore beg Leave to recommend him as a Person in all respects qualified for the Place he solicits.

"T. SIMPSON, F.R.S.
Member of the Royal Academy of
Sciences at *Stockholm*, and
Mathematical Master in the Royal
Academy at *Woolwich*."

December 5, 1757.

"Wm. MOUNTAINE
Mathematical Examiner to the
Right Worshipful Corporation
of the *Trinity-House*, and F.R.S."[5]

In this particular case, Simpson supplemented the printed form with a letter elaborating upon Mr. John Davis's qualifications:

"Dear Sir,

"You will perhaps be surprised at receiving a letter from me in relation to ye Subject I am now going to address you upon. Yesterday our worthy friend Mr. Mountain put into my hands a Letter from you relating to a Gentleman he had recommended to succeed Mr. Dodson. Permit me Sir to express my Approbation of ye prudent precaution which you observe in this matter and ye genteel manner in which you propose your doubts. Whatever Friendship or your Good-Nature may incline you to, Justice to your own Character, and a Regard to ye publick good ought certainly to have their due weight. But with respect to Mr. Davis I realy think him unfortunate in not being known to you, as I am positive that every part of his character will bear ye Test of a strict Examination. He was my pupil near two years

[5] This copy, in the Simpson collection, was sent to Sir George Oxenden Bart.; and many other copies were circulated among the gentlemen most concerned in making the choice to fill the vacancy.

(after being sufficiently qualified for a School-Master in ye Royal Navy and having actually discharged that duty with reputation). The elements of Geometry, Algebra, and a great part of ye Doctrine and Application of Fluxions (besides ye Demonstration of many things in Spherics, Dialling, Gunnery, &c.) were ye subjects he went through, and made himself very perfect in, *while under my Direction.* As to his moral character, it admits not of ye least doubt: He is a man of real Integrity and strict Decorum and seems incapable of those littlenesses which ingenious Men sometimes fall into, from a Want of a proper Knowledge of ye World and themselves. Should he succeed in his wishes, I make not ye least doubt but your Good-Nature will receive a sensible pleasure in having contributed to promote ye Welfare of a person who you will be convinc'd returns a strong sense of Obligations and who by his situation will have an opportunity to exert his natural abilities and hereby render himself still more an object of esteem & regard.

"I shall not trouble you Sir with an Apology for my having taken this Freedom: I know your disposition and have acted (to use your own words) just as you yourself would have had you been in ye same situation. I shall only beg leave to assure you that had you not declared yourself disengaged I would not have taken any step in the affair, and that I am with great respect,

"Sir, your most humble obedient Servant,

"Tho. Simpson."

"Woolwich Decr. 5. 1757."

This position, for which Mr. Davis campaigned was an important one, and reflected credit upon Simpson and upon his abilities as a private teacher.

John Turner of London, before mentioned, and Samuel Clarke were major contributors to the periodical *The Mathematician,* and both were at one time Simpson's students. Of the latter a draft of recommendation is to be found among the Simpsoniana:

"This is to certify that I look upon Mr. Samuel Clarke as a person not only well qualified to teach navigation and ye other common branches connected therewith, but also ye more difficult & exalted parts of ye mathematical sciences. He having acquired an accurate and very extensive knowledge in those subjects."

Mr. Edward Rollinson, "clerk to Mr. Hill in Park Street," was one of Simpson's students for several years, as well as a life long friend. He was able to utilize his mathematical training in a business way, and to gain a small reputation in answering and proposing problems in the *Ladies' Diary*. He became editor of the *Ladies' Diary* upon Simpson's death.

Alex. Buller of Philadelphia owed some of his mathematical training to Thomas Simpson and applied it in writing a very usable textbook on algebra, and in teaching mathematics in the American colonies.

In a letter to Francis Blake, Simpson mentioned a member of parliament "from Great Marlow in the County of Bucks," who was one of his pupils. Nothing further however is known of him, not even his name, except that he made "surprising progress" under Simpson's tuition.

Martin Folkes, one time President of the Royal Society, is said to have been one of Simpson's private students. He was in the habit of recommending to Simpson's tutelage certain of his friends, as the following letter would indicate:

"Sir—

I pray you will call on Wednesday morning soon after 9 o'clock upon Mr. Norden in St. James Street, at the blue periwig over at the Royal Bagnio who will talk with you on the affair I mentioned to you; I need not desire you will acquaint him with the terms on which you have usually assisted other Gentlemen in the same way, and I venture to assure you, you cannot be addressed to a more worthy person or who is more my particular friend, whom I have heretofore in the best manner desired to serve: you will send up word you come from me which will be sure to give you immediate introduction,

"I am Sir, very sincerely,
"Your very humble servant,
"M. Ffolkes."[6]

"Nov. 16. 1741."

No doubt many others to be numbered among Simpson's private students during his teaching career at London, later enjoyed successful careers as a result of his excellent tuition.

[6] Addressed to Mr. Simpson at the blew coat boy in long alley by Moorfields, London.

CHAPTER VII

Career at Woolwich

DELAY IN REMOVAL TO WOOLWICH

Although Martin Folkes advised Simpson to hasten his departure to Woolwich, there seems to have been some misunderstanding which caused a delay in his preparations. The following draft of a letter written by Simpson would seem to substantiate this theory:

"Sir

"Your manner of expressing your self, in your Letter to your Servant is ye Occasion of my presuming to trouble you with this, as it gives me all ye reason in ye World to believe either that he must have misrepresented ye Matter to you, or, that you are still unappris'd that my Lord Duke expressly signified his pleasure that I should remove to Woolwich as soon as possible. Whatever his representation of my way of expressing myself may have been all I said to him was that *as it was expected that I should enter upon ye Duty of my Post as soon as my warrant was delivered out which was to be ye next Tuesday, I should be very glad ye Goods cou'd be removed by ye Thursday or Friday following,* I had not at that time ye least Suspicion of any such great Inconvenience that might attend ye Removal as you have since represented and therefore could not think my request an unreasonable one which I confess it wou'd have been, as well as a piece of imprudence to run in haste to teach ye remaining part of a quarter without any Sallary or Reward (as I'm informed I must) unless I thought myself in danger of his Grace's displeasure and of disobliging Mr. Folkes by neglecting their Orders. This Sir is ye exact Truth and as such I hope it will entitle me to a better place in Your Opinion for ye future than you seem hitherto to have given me, if I may presume to guess from some unexpected Treatment I met with

at ye Board which I could not tell how to ascribe to anything but your influence.

"I am,
"Sir, with much respect,
"Your very humble and
"obedient Servant,
"T. Simpson."

"London Sep. 2. 1743."

"P.S. Mr. Graham, Sir, desires to give his humble Service to you, by whose advice I have taken this liberty of writing to you."

REMOVAL TO WOOLWICH

The next month, some time in October, found Simpson safely installed in his new home at Woolwich. Writing to Francis Blake from Woolwich on October tenth, Simpson says:

"Honoured Sir,

"I thank you sincerely for ye last Instance of your Goodness and Esteem, which could never have happened at a time when it would have been more acceptable. I am at last (Thanks to God and my Friends) safely removed to Woolwich, where I hope to do very well and, so far, like ye place extreamly. My Income here, reckoning everything, amounts to about 130£ p^r^. An. and my Teaching is only 3 Hours every other Afternoon; so that I still have an Opportunity of teaching in London two or three Days in a Week should I think proper to continue it; which I propose to do for some time, at least."

Simpson continues with a discussion of Gravity and of Mr. Folkes' honey comb problem which Blake had proposed to him in a previous letter.

"In attempting to explain ye Principles of Gravity you do well to exclude a Succession of Impulses; which, to account for ye thing, must either be supposed unequal in themselves or to be repeated after unequal Intervals of Time—Besides Gravity (whatever ye immediate cause of it is) is in all probability, a continued uninterrupted Force.

"Your last problem I have considered with a good deal of attention, but do not clearly understand your meaning, especially where you signify that, *from ye Nature of ye problem,*

and inspecting ye Cells, the four contiguous Sides EG, GB, BF, FD, must be equal, and different from AD & AE. For since the points *A, G, & F,* when ye triangular planes *ADE, EGB & BFD* are turn'd up, must coincide, in order to form a solid Angle I cannot see how *AE & EG* can differ or be unequal; it indeed appears plain that *AE & EG, GB & BF: FD & DA* must be equal, but that *AE & AD; EG & GB: BF & FD* should also be equal, is not so obvious. As ye Doctor was in Company when Mr. Folkes mention'd ye thing to you, I consulted him about it, and his opinion was that you had understood it wrong; for which reason I have not attempted y^e Solution nor do I intend it 'till I have waited upon Mr. Folkes, or you have given me some further In-

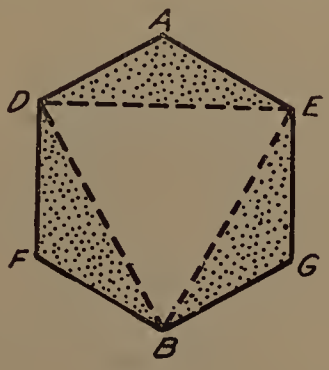


FIG. 17

formation of y^e Matter. I must not, however, forget to observe, that y^e Method you lay down for y^e Calculation is right, and will be of use tho' ye problem should prove different from what you took it to be."

Following the usual custom, Simpson concludes by proposing a problem for Blake to solve:

"Problem:

"Supposing *ACB* to be a Ballence or Leaver whose Brachia *AC & CB* do not lye in ye same Right line, but make a given angle *ACB* with each other; to find ye angle which each Brachium must make with ye Horizontol Line *CD*, so that 2 given Weights *F & G*, suspended at ye Ends thereof, may remain in equilibrio; the Lengths of ye Brachia being given.

"If when you go to Newcastle next, you would be so good to

desire Mr. Brysan to return ye Books of Annuities which remain unsold it will add to ye many obligations confer'd upon,

"Honored Sir,
"your most obliged &
"most obedient Servant
"T. Simpson."

"Woolwich October 10. 1743."

Thus October, 1743 finds Simpson established at Woolwich Military Academy as Second Master, a position which he regarded as a decided rise in station, and one which meant a very satisfactory income. His intentions, as he himself stated, were to continue with his private teaching in London for a time at least, and to continue his teaching in correspondence with such men as Blake in order to fill the time which he anticipated "would hang heavily" on his hands. Coming down from London where he was accustomed to much company, to the country where he was limited to the society of boys and a few masters, must have required considerable adjustment on his part.

NATURE OF THE INSTRUCTION OFFERED AT WOOLWICH

The Royal Military Academy at Woolwich was the only one of the three military schools England has to-day, which was in existence during the eighteenth century. It was established in 1741 by George II, to fill a need under which the English army then suffered, by giving special instructions to officers and men who served in the Artillery and in the Engineers. Its beginnings were of the humblest sort imaginable. A single room in a house at Woolwich, where the Board of Ordnance used to assemble occasionally, was set apart by the Government as a hall of study; and two masters were appointed to give lectures by rotation, during four consecutive hours, in three days of every week. John Muller was appointed head-master at a salary of 200£ a year by the new Master-General, John Montagu, second Duke of Montagu. At first only the officers of the single battalion composing the English Artillery and of the corps of Engineers were required to attend.[1]

Later the room was thrown open to the non-commissioned

[1] *Encyclopaedia Britannica*, XII ed., "Academies, Military."

officers and privates, and eventually to the cadets. The cadets, however, being the sons of officers of the corps, proved very difficult to manage, for they were neither dressed in uniform nor under military control. The masters resented military interference and the boys defied the masters at will.[2]

For this reason, in 1744, the cadets were first clothed in uniform and collected into a distinct company. Two officers with a drum major undertook the management of them, and the arrangement worked, or was supposed to work so well, that the numbers composing the cadet company were gradually increased. In 1782 they had grown in number from twenty to sixty.[3] After this arrangement attempts were made to lodge and board the cadets as well as to educate and drill them.

It was here that Thomas Simpson and John Muller attempted an education in all the branches of mathematics thought necessary in preparation for military service. Muller had easily obtained his appointment as head master of mathematics in 1741, and later he was made professor of fortification and artillery. He held this post until he was superannuated and pensioned in September, 1766. Previous to his appointment to Woolwich, Muller had held a position in the office of the Ordnance, so that the transfer to Woolwich was relatively easy.[4] On the other hand, Simpson's appointment required the efforts in his behalf of such a distinguished friend as Martin Folkes, when in 1743 Mr. Derham's death occasioned a vacancy. A great deal about Simpson's career and about the school can be learned from Simpson's private notes and correspondence. Among them occurs the following outline of the curriculum:

"*Directions for ye Royal Academy Masters at Woolwich.*

"No boy to be admitted under 14 or 15 year's of age, nor even then unless he is master, at least, of ye Rule of Three.

"The second master is to teach and perfect them in Figures, teach ye principles of Algebra and Elements of Geometry, by his own method, and continue his Instructions untill they have made a sufficient progress to begin with ye First Master, who is to examine them before he proceeds.

[2] Duncan, *History of the Royal Artillery*, vol. 1.

[3] May, 1750, Simpson had 36 in his class, while in September, 1757 he had 44 in his class of gentlemen cadets as his private notes will show.

[4] *Records of the Royal Military Academy.*

"The first Master to teach Trigonometry, conic-Sections, practical Geometry, Mechanics, Gunnery, and Fortifications (with ye Addition of Surveying, Mensuration and Levelling for those designed for Engineers) ye Construction of Guns, mortars, machines, Instruments, and Tools relating to Artillery, to take dimensions and Plans of them, which are to be drawn in a Fair-Book by ye Pupils themselves, as shall everything else they learn of ye First and Second Masters, in order to be produced upon their examination or when called for by proper authority.

"If it should be thought necessary for Mr. Muller & Simpson to teach only three half days in a Week, Mr. Muller shall give them sufficient employ for his intervening days, which with proper directions from him Mr. Simpson will inspect (supposing them out of Mr. Simpson's class). Mr. Simpson shall task his class for Mr. Muller's Days, who will likewise take care of them during his attendance.

"The Drawing master before he begins on Landscapes is to teach them to draw lines, Guns, Carriages &c. by Scale, and then perspective which will better prepare them for Mr. Muller's Instruction, and as an addition for ye Engineers Branch, ye Method of Sketching Ground.

"The French Master is to teach as usual.

"Everything relative to ye Attack and Defence of places, making Batteries &c. will be taught them when ye polygon is erected.

"During ye gun and mortar Exercise a certain number that have made a tolerable proficiency in ye Academy, to attend and assist in proportioning ye Length and cutting the Fuzes for ye mortars, and enter ye Remarks in a Book.

"All such that shall be judged sufficiently versed in ye Theory are at a certain times to attend & work in ye laboratory, to be instructed by ye chief toast masters or his Assistant, in mixing of Composition, proving and fixing Shells, Driving Fuzes, making grape shot, and everything else for ye Service of ye Artillery.

"The cadets to be publicly examined every quarter, beginning at Michaelmas next, and each master to sign a report of ye progress ye Cadetts make in their respective branches."

Simpson gives a further account in a letter to the Master-General of the Ordnance, written January 28, 1760, in which he

has commented on these various articles. His remarks furnish an insight both into the nature of the work given at the Academy and with regard to Simpson as an educator.

"Observation on Article I.

"If ye order were that no lad be admitted unless He be perfect in ye double rule of Three and know something of Fractions; such a regulation would I apprehend be of Benefit by excluding lads of no capacity; and would also be a means to ease great men of troublesome applications with which they are sometimes unseasonably pressed. Were every lad in order to his admission to be examined by all ye Masters in whose Branches he has made any proficiency (instead of ye first master only) that would, certainly, be a means to prevent partial Representations.

"On Article II.

"The Second Master, after having given any pupil a proper Degree of Knowledge in ye Branches of Theory here assigned to him may I conceive be allowed to continue to his instructions to such pupils in ye practical application thereof, in whatever manner shall appear to him most conducive to ye forming of a good engineer or an Able Artillery Officer; provided that He do not interfere with Y^e^ First Master, in ye Branch which he is at that Time teaching to ye said pupils.

"On Article 3.

"This article seems capable of many improvements. For, in ye first place, the conic sections, which take up much time and are very difficult to learn may very well be omitted; being only put down as necessary to Y^e^ Theory of Gunnery; which, at this time, is better understood and demonstrated without them.

"Secondly. Practical Geometry (whose use and design seems not to be well understood) is also improperly placed here; being included in ye branch of ye Second Master. For, the drawing of lines, making of angles, and ye constructing of various kinds of Figures are not only taught, but demonstrated in ye Elements of Geometry; to which they properly belong. This part of ye Theory from its great use in ye common concerns of Life has, among artificers and mechanics (who learn from practice, and are not solicitous about demonstrations) acquired the name of practical geometry.

"Thirdly.—The Use of the Globes and ye study of Geography,

tho' very useful to military Gentlemen is here intirely omitted. As the first Master has never yet made this Branch a part of his duty, I have great cause to think that He would not be sorry to see it enjoined to ye Second Master.

"Fourthly.—If in ye teaching of Fortification, the use of Models were to be enjoined, and the manner of attacking and defending fortified places were to be shewn thereby it would be of great benefit: By this means those who have not a capacity for geometry and difficult Calculations, might acquire some degree of knowledge in that necessary Art, for a common sufficient Artillery officer. Whereas the learning of fortification by merely drawing lines upon paper (ye only method now practiced in ye Academy) affords but very imperfect notions to young minds.

And in confirmation of this opinion, some officers, who once attended ye Academy, have assured me that they had, since, gained more reel knowledge of this art in the space of a month, by attending ye common lectures in Town from proper models, than they had before learned during all ye years they continued at ye Academy. A Testimony very little to ye honor of a Royal Institution! A model designed for ye Academy is indeed in ye possession of ye First Master; but so far has He been from making ye proper use of it that he even (I am told) refuses a Sight of it to all, except his particular favourites. . . . The Globes and other Instruments sent down for ye Academy (to the amount of many score pounds) have not been one bit better employed. And I really believe that could ye present condition of those Instruments be known, a scene of bad management would present itself.

"On Article 4.

"This Article (if I understand it right) cannot I humbly conceive, be productive of any good effects; but seems rather to have a Tendency to create confusion as things are now circumstanced. The subjecting ye second master to ye direction of ye first would not be of advantage to ye Institution: and to make the first master depend on the second would be irregular. This paragraph would run better, and be equally to ye purpose, if ye words "which will better prepare them for Mr. Muller's Instructions" were to be expunged.

"On Article 10.

"I am convinced from Experience that this regulation will be

found of great Utility whilst merit and a diligent discharge of Duty are countenanced by Great Men, and made necessary steps to Preferment.

"I have sir delivered my sentiments with great Freedom. . . . I shall esteem myself exceeding happy in having contributed something to ye Good of an Institution under which I am placed, and of having, on this occasion, ye Honour to subscribe myself. . . . "

Although these comments upon the proposed regulations shed a great deal of light upon the curriculum of the Military Academy it must be born in mind that Simpson was in constant enmity with the first master, John Muller; consequently his recommendations are somewhat biased.

ENTRANCE EXAMINATIONS

Admittance to the Academy was usually obtained through the master-general of the Ordnance, and he in turn sent the candidates to be examined by the masters of the Academy. Several testimonials as to the ability of the various candidates are to be found among Simpson's private notes, of which the following is a fair example:

"Honourable Sir,

"In obedience to your commands, I have examined Mr. J. Rigault in Figures, and find that He knows as far as ye Rule of Three and is pretty ready in that. And I declare farther that, in this respect, he is better qualified than some that have lately been admitted into ye Academy, as Cadets.

"Thomas Simpson."

"May 25, 1759."

The following letter serves to show that Woolwich Military Academy was open to only certain types of people:

"To Mr. Jos. Barrett Shoemaker in Nottingham,

"Sir:

"I am sorry that you have been at ye Trouble to apply to me for Information that will not turn out answerable to your wishes.

"The only Thing in ye Artillery that would suit your Son, and to which he could have any sort of pretensions, is ye post of a gentleman cadett: These are the persons whom it is our business

to instruct; (in order to their being qualified for Artillery Officers or Engineers) and amongst whom none are admitted but gentlemen's Sons (or such, at least, as appear under that Rank):

"As it is therefore intirely out of my power to serve you (unless this objection could be got over by means of a powerful Friend) I must desire you to desist from ye further Trouble you purpose to give yourself in sending to me a specimen of ye Youth's performance, as it would answer no manner of purpose.

"I am Sir,

"Your very humble servant,

"Tho. Simpson."

"Woolwich, Sep. 3. 1760."

SIMPSON AS A MASTER IN THE ACADEMY

Simpson seems at first to have entered into the spirit of his new work with great enthusiasm. Hutton, who is the best authority on Simpson's personal approach to his teaching, says that "his manner of teaching was fine; he had a peculiar and happy address; a certain dignity and perspicuity, tempered with a degree of mildness, as engaged both the attention, esteem and friendship of his scholars; of which the good of the service, as well as of the community, was a necessary consequence. . . . It must be acknowledged however that his mildness and easiness of temper, united with a more inactive state of mind, in the latter years of his life, rendered his services less useful; and the same very easy disposition, with an easy and unsuspecting simplicity, and playfulness of mind, rendered him often the dupe of little tricks of his pupils. Having discovered that he was fond of listening to little amusing stories they took care to furnish themselves with a stock; so that having neglected to learn their lessons perfect, they would get around him in a crowd, and, instead of demonstrating a proposition would amuse him with some comical story, at which he would laugh and shake very heartily, especially if it were tinctured with the somewhat ludicrous or smutty; by which device they would contrive to wear away the hours allotted for instruction, and so avoid the trouble of learning or repeating their lesson. They also tell of various tricks that were

practised upon him in consequence of the loss of memory which he experienced in the latter part of his life."[5]

Hutton's description of Simpson as a mild tempered man is difficult to reconcile with the many disputes he had throughout his life; and a great deal of what Hutton has to say about Simpson's teaching at Woolwich was a repetition of traditional anecdotes and schoolboy yarns, which in the telling may have suffered exaggeration.

The earlier years of his teaching at Woolwich were without doubt very profitable and he not only instructed his pupils at the Academy thoroughly but also others whom the superior officers of the ordnance "permitted to be boarded and lodged in his house."

[5] *Memoirs of the Life and Writings of the Author*, by Charles Hutton, printed as the preface to *Select Exercises for Young Proficients* by Thomas Simpson, 1792 ed.

CHAPTER VIII

Publication of Textbooks and Election to the Royal Society

PUBLICATION OF A TEXTBOOK ON ALGEBRA

While engaged in guiding Randall's activities in 1744 and seeing his treatise to press, Simpson was also busy in preparing a textbook for publication. During his career at Woolwich his works took on a different character. Owing to the fact that he was instructing youth in algebra, geometry, artillery, and kindred subjects, it was natural to find him preparing textbooks on these subjects.

Consequently in 1745 he published at London a work entitled, *A Treatise of Algebra; wherein the fundamental Principles are fully and clearly demonstrated, and applied to the Solution of a great Variety of Problems, and to a Number of other useful Enquiries.* To this text is added *The Construction of a great Number of Geometrical Problems, with the Method of resolving them numerically.* The work was expressly designed for the use of beginners.

The brevity of Simpson's text as compared with contemporary works by Saunderson[1] or Hammond[2] is immediately apparent. In addition to the fundamental operations, he gives Sir Isaac Newton's method of divisors, Cardan's solution of the cubic, Descartes's solution of a biquadratic, and Newton's binomial theorem. Descartes had succeeded in effecting a simple solution of problems of the type,

$$x^4 + px^2 + qx + r = 0,$$

[1] *The Elements of Algebra in Ten Books*, by Nicholas Saunderson, Cambridge, 1740.

[2] *The Elements of Algebra, in a New and Easy Method, with their Use and Application in the Solution of a great Variety of Arithmetical and Geometrical Questions: By general and universal Rules. To which is prefixed an Introduction containing a Succinct History of This Science.* By Nathaniel Hammond of the Bank, London, 1742.

a method considerably improved by his commentator Van Schooten (1649). Simpson brought the method into modern form in this work.[3]

Among the practical applications of algebra, Simpson includes a chapter on simple interest; one on annuities and pensions in arrear; a discussion of compound and simple interest; the investigation of several theorems for the solution of the various cases of compound interest and annuities; and finally a discussion of plane trigonometry. Simpson has given the solution of equations by infinite series, a favorite topic of the eighteenth century.

At first he intended to include a discussion of the doctrine of infinite series and the application of algebra to the higher geometry of curves together with the linear construction of problems of several dimensions, but in order to limit the size of the textbook he was obliged to omit these features. As it was, the textbook writers of his time were more influenced by a desire to include in their works all subject matter that could be referred to under the heading of "algebra" than by a desire to produce a clearly presented explanation of what may strictly be termed the elements. In subsequent editions of Simpson's work, illustrations of the rules are given in a footnote arrangement in the form of "demonstrations." These demonstrations are complete explanations of the rules. This practice of explanation by footnote however led to a general acceptance of rules by students without a complete understanding of their meaning.

Simpson's *Algebra* was popular. It was used to a great extent in the American colonies,[4] as well as in Great Britain. The first American edition was a reprint from the eighth London edition, and was printed at Philadelphia for Mathew Carey by T. & G. Palmer in 1809. The second American edition appeared in Philadelphia in 1821, printed by David McClure, teacher of mathematics.

Acquaintance with William Jones. Simpson dedicated the textbook to William Jones, Esq., F.R.S., who was himself the author of the *Synopsis*, and among other things vice-president of the

[3] Smith, *History of Mathematics*, II, 469.

[4] It was advertised in the *Virginia Gazette* September 17, 1772, but was in use much earlier.

Royal Society at London for a time. His friendship with Simpson was one of long standing for, as the dedication would indicate, he seems to have encouraged the preparation of this book:

"Sir,

"From the time you was pleased to signify that notwithstanding the many books of Algebra already extant, something on that Head well executed, might still be of use, I had a strong Inclination, however unequal to the Task to attempt a Work of this Nature; and therefore as the following sheets owe their birth to You, I think were it For that Reason alone I might claim a Sort of a Right to address them to You; than whom no Man living is a better Judge both as to what has already been done and what is further necessary on the subject.

"I cannot so far flatter myself to think I have come up to that Accuracy and Comprehensiveness for which You are so universally distinguished, and, which I have endeavoured to imitate yet hope you will meet with some things here not altogether unworthy your Approbation, and, which, with greatest Respect and Deference, are humbly offered to your Protection, By, Sir, Your most Humble and most Obedient Servant, T. Simpson."

Simpson seems to have consulted with Jones on different occasions concerning his researches, for among his papers occurs a letter addressed to William Jones containing an explanation of "Fluents of Multinomials and Series affected by radical Signs, which do not begin to converge till after the Second Term." This letter, or paper, was no doubt read before the Royal Society at one time, because something of a similar nature was printed a few years later in the *Transactions*,—January 10, 1750–1.[5] This friendship brought Simpson a great deal of good fortune.

Additions to the Algebra. Additions and improvements to the *Algebra* appeared in the second edition of 1755, among which is

[5] He writes, "What gives me ye greatest encouragement that this little Essay will meet with your Approbation is that it is not merely an abstracted useless speculation but may be apply'd to good purpose in many difficult and important Enquiries into nature; whereof I have given one or two instances; and shall further take ye liberty to observe here that most of ye Lunar Equations delivered by Sir Isaac Newton, are nothing more than such approximations as may be exhibited by ye first Term, only of a series derived by the Method here offered to your perusal."

"a new and general Method of resolving Biquadratic Equations that are complete, having all their terms." This edition was dedicated to James, Earl of Morton, F.R.S., Mr. Jones being then dead.

Influence of the Algebra on Later English Writers. While no less than eight editions of the algebra were published during the eighteenth century, the work was also copied by many of the later writers. The *Elements of Algebra* by Cadell, published in 1779, was a work meant by the author for use in the university, but he has borrowed freely from Simpson. Bonnycastle, in the preface to *An Introduction to Algebra*,[6] states that he has attempted to take the best from the works of Newton, Maclaurin, Saunderson, *Simpson* and Emerson, as well as making some recognition of continental writers.

Correspondence with Regard to Algebra. After the publication of his *Treatise of Algebra*, Simpson received several communications showing the contemporary opinion of it. The opinion as expressed by one of his public reads as follows:

"Your Treatise of algebra has afforded me great satisfaction as it contains many things both useful and entertaining some of which are wholly new, and others are exhibited in a clearer light, and demonstrated with more conviction to the understanding than I have met with in any author. It is therefore with great deference to your superior ability that I venture to communicate to you a few observations on some parts of your book, which if I am not mistaken, are capable of amendment. Those relate chiefly to your geometrical constructions and Arithmetical solutions of Problems; a subject which I confess you have treated more fully, and in many cases more fully illustrated of than any author I have seen. Yet I cannot but think that some of your constructions and demonstrations, are more complex and embarassed with more lines than are necessary, and some of your calculations require the use of the Trigonometric Canon, where the problems may be easier solved without.

"To show this is not a groundless assertion I have pointed out three or four problems, which I have attempted to solve differently, and as appears to me, at least as to two of them, in an easier and simpler manner than you have done. It were want of

[6] *The Monthly Review*, vol. 69, p. 265, contains an account of this work.

candour in me not to acknowledge that I have failed in attempting better solutions of others of your problems, particularly of the 76th and 77th. . . . "

Another letter, from Caleb Burton of Sandwich in Kent, dated July 13, 1746, gives a fair illustration of one type of audience Simpson reached through the textbook outside of the Academy.

"Sir,

"Being an entire Stranger to You, I don't know how to excuse my giving you this Trouble, which, what I am about to say, I doubt, will not.

"I have a great Inclination to Mathematical Knowledge, but at the same Time have the . . . unhappiness of being Situated where I have not Opportunity of getting Any but by the Assistance of Books, which have been my only Teacher in what little (indeed very little) Knowledge I have got in those Sciences. Among other Books, I have for that purpose bought your Treatise of Algebra, which has given me a clearer knowledge by much in that Art than I had before; but yet for want of Capacity of Apprehending you, or a knowledge Sufficient for that purpose, I have met with a Difficulty I can't get over tho' I have endeavour'd at it a long Time, because I was unwilling, as I knew I had no right, to give you this Trouble. And that is the forming the Theorem

$$e = \frac{g + wg^2}{1 + sg + tg^2}$$

for extracting the Roots of Equations, in Page 152 of your Treatise.

"Now Sir, as I dont doubt but that you publish'd that Book with the generous Design of instructing the Ignorant, and as you don't seem to me to Deliver that Theorem as a secret, tho' I cannot comprehend the formation of it, I hope you will excuse me tho' unknown If I beg of You the Explanation of it. After this I don't dare to be so troublesome as to beg the Demonstration of the Lemma Page 166 of your said Treatise. The work of this Lemma I have met with elsewhere but in a much less neat and elegant Method than Yours. . . . "

At this time Simpson was called upon in his correspondence

with Blake to justify certain parts of the work. His treatment of negative quantities seem to have been Blake's chief cause for objection. Thinking of algebra as a "species of arithmetick," Simpson had explained the use of the signs + and − as signs of operation, but he had introduced without explanation their use as symbols for both positive and negative numbers. This was a common procedure among the textbooks writers of the period, and can be found in practice in the works of Clairaut, Maclaurin, Emerson, Trail, Bézout, Sauri, Blassière, Büsch, Bossut, and many others.[7]

Simpson could not clearly discover a new kind of number which would generalize subtraction; this may be seen from his remark to the effect that "$-a$ is in a sense as impossible as $\sqrt{-b}$, since it is impossible to subtract a from nothing; the idea or the illusion about quantity less than nothing is meaningless." His treatment of the product $(a - b)\ (c - d)$ most nearly resembled that of the French author Clairaut,[8] although he does not carry his thinking as far. Both authors take the product $(a - b)\ (c - d)$ by multiplying first $(a - b)$ by c, $(a - b)$ by d, and by subtracting the second product from the first.

Simpson's ideas on this subject most clearly appear in a letter written in answer to some objections offered by his friend Francis Blake:

"Honoured Sir,

"I was ill when yours came to hand, and this is y^{e} first time I have put pen to paper since I was sensibly concern'd to hear you were again tormented with that painful Distemper, but am persuaded it is rather *owing to a strong Byas for Old Opinions* than y^{e} *Gout*, that you appear a little sanguine in your Attack upon my Book,—whether I have raised ye Devil without well laying him again or whether my reasoning be not so general as I apprehend I shall not here Dispute, but only beg ye Liberty to offer ye two or 3 following particulars to your consideration.

"First, as pure Algebra, or Algebra considered in itself, is a species of Arithmetick, whose Subject is Abstract Number, wherein we can have no Ideas of Position, but of Magnitude only,

[7] Cantor, *Vorlesungen über Geschichte der Mathematik*, IV, 82.
[8] Cantor, *Op. cit.*, IV, 84.

I cannot conceive how we *can have Ideas of negative Quantities, unless* we can *suppose* these are *Quantities actually less than Nothing, which you will hardly allow.*

"Secondly, it *does not seem consistent* with perspecuity and y^e^ Rigour and Accuracy of Science to *ground a Subject,* perfectly *abstracted upon foreign and mechanical considerations of Debtor and Creditor,* and upon cases such *as multiplying* $-a$ *by* $-b$, which *can never happen in Nature or practice,* not, even after y^e^ problem is brought to an equation if y^e^ Operation be rightly managed. And if this be true, why is it not reasonable to think that y^e^ Difficulties so universally complained of with regard to y^e^ signs, in Multiplication and Division of Algebra are not properly such in themselves but only made so by y^e^ Manner which Authors, one after another, have treated them. Lastly, I could wish to know what Idea you have of y^e^ Quantity $-12ax$, as to its *Value,* and should also be glad if you would *produce one single Instance in Nature or Practice where it is required to subtract such a Quantity.*

"I am not yet a Fellow[9] of y^e^ Society, nor am I fond of that Honour, upon account of y^e^ Expence,[10] tho' Mr. Jones[11] persuades me to it and tells me I shall find my advantage in it. When I was with him, about a month ago, who should come in but ye Doctor [Doctor Bevis], (whom I had not seen for 9 months before) with a chairman's blew coat on, hanging down to his Toes, a very dirty shirt, and his Face all over Scratches; I could see Mr. *Jones* received him very coldly, but I left him there— I have no News worth writing, unless what relates to the bad success of our Armies abroad,[12] which has put every body here in consternation; and which you will hear before this comes to hand. However, let what will happen, I shall always deem it a great satisfaction to have the honour of subscribing myself, Honoured Sir, . . .

"When I am one of the Society, if ever that should be, there is nothing I should more wish for there than your company; and,

[9] He was made F.R.S. the next year, in 1746.

[10] When he became F.R.S., he was excused the payment of fees.

[11] Mr. William Jones (1675–1746), hitherto mentioned as author of the *Synopsis,* and as a fellow of the Royal Society.

[12] England was at war with France and Spain, and met defeat in 1745 from the French under Marshall Saxe, at Fontenoy, in the Netherlands.

if you would condescend to become a Member too, I am persuaded it would afford a great many opportunities of Gratifying an inquisitive Taste."[13]

The correspondence between Blake and Simpson during the next two months is concerned with Simpson's inability to follow Blake's treatment of converging series; but Simpson was rather slow to answer Blake's communications for he thought that the troublesome affairs in the north with the "Young Pretender's" defeating of the British at Prestonpans, would leave Blake little leisure or inclination to study mathematics. In October Simpson wrote Blake,

"I am very glad ye King's Troops are so far advanced and begin to condense towards ye North. There is now no longer Danger of ye Rebels making you a Visit,[14] which I was once under some Apprehensions of. People here begin to be very Impatient for ye Event and never was a spirit of greater Loyalty and Zeal discovered upon any Occasion, which must be at this time a singular satisfaction to every one, whose principles are opposite to popery, as those of, your most humble & obedient servant. . . "

From this time on correspondence between Simpson and Blake, as far as can be judged, became more and more irregular. Simpson became pre-occupied with his work at the Academy, and Blake found more of his time being monopolized in business. The irregularity of their correspondence from this time on also indicates that Simpson found it impossible to arrive at an understanding of some of Blake's ideas regarding fluxions and fluents. In a discussion with respect to the "expression $x\dot{y} + \dot{x}y + \dot{x}\dot{y}$,— and what it represents before $\dot{x}\dot{y}$ is rejected," Simpson wrote:

"But (I say again) in a Method like yours that has its whole foundation upon uniform or (if I may so speak) imaginary Motion, I cannot see your Reason for introducing accelerated distances (except it be y^{e} Fluents themselves).

"Are $\dot{x}$ & $\dot{y}$ to be considered as actually generated by such velocities as the relation of the fluents requires, or as the spaces which might be uniformly described, with the invariable velocities

[13] Letter written by Simpson, July 4, 1745.

[14] The Rebellion of 1745 seriously affected the activity of the Philosophical Society of Edinburgh, and it was really a decided check to the development of science.

at points *a* and *d;* If the former, then the expression holds true; but this makes it apply merely to a particular case, etc. . . . "

Such arguments finally resulted in a deadlock of ideas and as Blake does not appear to have published a reprint of his essay or anything else on the subject of fluxions, it would seem that this inability to arrive at conclusions agreeable to Simpson discouraged him from further effort.

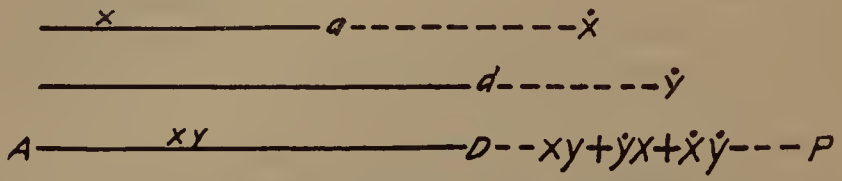


Fig. 18

ELECTION TO THE ROYAL SOCIETY

Simpson's remarks to Blake concerning his possible admission to the Royal Society, and his reluctance owing to the fees, is of interest since he became a member late in 1745. Hutton says, "The president and council, in consideration of his very moderate circumstances, were pleased to excuse his admission fees, and likewise his giving bond for the settled future payments." Hutton gives the date 1745 as the year of his admission, saying he was proposed as a candidate by Martin Folkes, president, William Jones, George Graham, and John Machin secretary, all very eminent mathematicians.

Rôle of Learned Societies in the Progress of Mathematics during the Eighteenth Century. To understand the full extent of the honor conferred upon Simpson when he was elected to the Royal Society, it is necessary to realize what an important part societies played during the eighteenth century in the development of mathematics and mathematical education. During this period each of the two opposing types, the Royal Society of London and the Philosophical Society of Edinburgh on the one hand, and the mathematical clubs which were to be found throughout England on the other, had a unique influence upon the development of mathematics. The former tended to recognize only mathematicians of a high level of attainment, while the latter popularized mathematics among the less intellectual classes.

Simpson's early life in London is representative of the type of activity associated with the mathematical clubs. Such social organizations composed of members who during the day were engaged in industrial and commercial pursuits, were seldom distinguished for the high scientific attainment of their members. The journals to which members of such clubs contributed such as the *Ladies' Diary*, *Merlin's Magazine*, or the *Gentleman's Magazine*, generally published an elementary type of problem material. Even the mathematical disputes between members of these organizations as is evidenced in Simpson's early life were often of such an ordinary nature that Simpson felt called upon to remark, "because they cannot rise to ye Level of Others, they industriously endeavour to bring others down to their own level; who like Maggots in ye richest cheeses are always sticking closest about ye least finished parts of good authors." Yet such clubs did provide educational advantages to a mass of people whose interest in mathematics was fired by the greatness of Newton. As an educational factor these clubs are not to be disregarded.

On the other hand, the Royal Society of London was founded in 1660 by a group of men who were known as the "invisible college," and who were characterized by their high intellectual attainments. Such names as Halley, Newton, Jones, DeMoivre, Machin, and Taylor, of the early eighteenth century were replaced in Simpson's time by the most outstanding of his contemporaries, such as Maclaurin, Stewart, Robert Simson, Landen, and Blake. The brilliant group of mathematicians on the list of membership found in Newton's time was not duplicated after his death. The Society was not as influential as it might have been after 1730 and did not rank as high as did the Academy of Sciences at Paris. The reasons for this were obvious. The Royal Society at London struggled along without government aid, while the Academy of Sciences was richly endowed. Membership in the Academy of Sciences was limited, while elections to the Royal Society were on a liberal scale.

Yet the Society served a distinctly useful purpose during the eighteenth century. The British Museum was not created until 1753, and meanwhile the Royal Society's repository was the center to which specimens of every kind were sent from all parts of the country and from abroad. This collection was very large

when, in 1779, it was given to the British Museum. The Society was also the most effective means of communication in regard to mathematical questions between England and the continent at this time. The foreign correspondence of the Royal Society was such as to keep the foreign secretaries well occupied.[15]

The ease of membership was a frequent subject of complaint. The charter book for 1730 states that every person to be elected a Fellow shall be "propounded and recommended at a meeting of the Society by three or more members" and shall also deliver a paper signed by himself specifying his name and chief qualifications. Yet a member is said to have remarked that a man was surer of election if he had done nothing of merit, for then the value of his work would not be a subject for argument. More stringent measures were taken by 1743, as is evidenced in a letter from Thomas Simpson to Francis Blake before he was yet a Fellow:

"As to ye privilege of being admitted a fellow to ye Royal Society, it is not so easily obtained now; as it was some time ago; for, then such members were entered without any sort of merit, that ye Society (as I am informed) have found it necessary to come to a resolution not to receive any person as a member who has not first distinguished himself by something curious. But this is no Barr to you since ye Book you have wrote (or anything you may publish in y^{e} *Transactions* hereafter) will be sufficient recommendation for you; would your modesty but permit you to own it."

The high admittance fees and yearly dues did a great deal toward maintaining the Society in some measure of exclusiveness, but at the same time genius is not always accompanied by wealth, as in the case of Simpson himself. Notwithstanding the fact that the Society was so pressed for funds in 1741 that its president, Martin Folkes, gave 100 pounds to assist "in the low state of the revenue," when Simpson was admitted he was excused payment of fees and dues. Unfortunately, the Society could not make a general practice of this procedure even in deserving cases. Nevertheless the fact remains that the most prominent mathematicians of the later Newtonian school as a

[15] This office was held by Dr. Stack in 1748; by Dr. Parson in 1751; by Dr. Maty in 1762; and by Dr. Bevis in 1766; and by P. Maty in 1772.

rule belonged to the Royal Society, and the Society was representative of the best that England had to offer. It is a fault of the century and not of the standards of the Society that the achievements of the Fellows who were of the later Newtonian school were in no way comparable with those of Newton's immediate time.

After Simpson had become a member, numerous people wrote him asking the proper method of procedure in order to gain admittance. To one he gave the answer that "it is hardly practicable without a very strong Recommendation, as you have not yet published anything whereby Those acquainted with you can form a just Idea of your Merit and Abilities." To such a man as John Landen,[16] however, Simpson wrote encouragingly:

"If you cannot bring yourself to speak to Dr. Smith[17] on that head (to whom I should take the liberty to write were I not wholly unknown to him) you need only, when Business brings you next to London, contrive to spend one evening with me and a few of my Friends (who are members); and then I will answer for it, that ye thing shall be done, without farther trouble. To introduce a person of your character and Abilities into ye Society, is rather conferring and Honour on one's self, than an obligation on ye person recommended."

Landen did not take advantage of this opportunity, for this letter was written in 1758 and he did not become a member until 1766.

The foreign membership was in no way more exclusive than was the resident membership. While the Academy of Sciences at Paris had only eight foreign members, the Royal Society felt that it had taken stringent measures in 1765 in limiting its number to eighty.

In spite of these handicaps the Royal Society conducted extensive experiments in connection with the observatory at Greenwich, and material decidedly superior in character to that found in the popular journals was published in the *Transactions of the Royal Society*. In 1753 a committee was appointed to select from the articles presented those worthy of appearing in print.

[16] Smith, *History of Mathematics*, I, 457. See below, Chapter XII, pp. 176–181.

[17] Dr. Robert Smith (1689–1768), Plumian professor of astronomy, Trinity College, Cambridge. See Smith, *History of Mathematics*, I, 446.

Thomas Simpson had seven papers published, but most of these he later printed in one or another of his treatises.

The only other scientific society in any way comparable to the Royal Society, was the Philosophical Society of Edinburgh, founded in 1739. Although this society was founded in the interests of science its progress was somewhat retarded by the Rebellion of 1745 and the death of one of its founders, Colin Maclaurin, in 1746. In a letter to Stirling, February 4, 1738, Maclaurin includes a postscript concerning this organization:

"P.S. I forgot when you was here to tell you that last spring some gentlemen had formed a design of a philosophical Society here which they imagined might promote a spirit of natural knowledge in this country, that you was one of the members first thought of, and that Ld. Hope and I were desyred to speak to you of it.

"I hope and intreat you will accept. The number is limited to 45, of which Lds. Morton, Hope, Elphenston, St. Clair, Landersdale, Ld. president and Minto, Sr. John Clerk, Drs. Clark, Stevenson, St. Clair, Pringle, Johnston, Simpson, Martin, Mes. Munroe, Craw, Short, Mr. Willm. Carmichael &c.

"I shall write you a fuller account afterwards if you will allow me to tell them you are willing to be of the number."

With the exception of Robert Simson, and Short, the names mentioned are hardly well known in a mathematical way. The society, however, continued to exist until 1783, when it became the Royal Society of Edinburgh in which Hutton and Playfair took an active interest.

It can be seen that membership in the Royal Society of London was regarded as the highest possible recognition of ability. Simpson accordingly gained in reputation when he became a Fellow. Association with others who had received such national recognition meant the encouragement of his most creative abilities.

ILLNESS

Owing perhaps to overdiligence in the preparation of books and in teaching, Simpson found it necessary to take a trip to Market Bosworth in 1747 and place himself under a doctor's care. His illness must have in some measure interfered with his duties at the Academy. A letter written by Simpson from

Market Bosworth on January 28, 1747 gives an account of his condition:

"Most Honoured Sir,

"After ye most sincere and grateful Thanks for your new assurances of Friendship and favour, I beg Leave to acquaint you that according to your Directions I have tryed every method that appeared most conducive to ye restoring of my Health. I have had both Sr. Richd. Maningham's and Dr. Munro's assistance and also that of a very eminent physician here in ye Country (where it was thought proper I should retire for ye Benefit of ye Air) all which I hope, have had and soon will have ye desired effect. I find myself considerably better that I can exercise a little more to recover my perfect health.—I hope you will pardon this Scrol and believe that I shall be impatient 'till I return again to my Duty, and that it will always be my chiefest concern to so preserve ye place I have ben so happy as to gain in your esteem and, to shew myself not altogether unworthy of ye extraordinary Favours confer'd upon,

"Honoured Sir,
"your most obliged and most obedient
"Servant,
"T. Simpson."

"P.S. If his Grace should happen to speak of me, vouchsafe to let him know that I most humbly beg Leave to return Thanks for his great Forbearance and Goodness to me in my Afflictions and that I hope to be soon able to return to my duty being so much better."

While Hutton speaks of the climate at Woolwich as extremely unhealthful, there is no doubt that Simpson's diligence in the preparation of textbooks, combined with his enthusiastic performance of his duties at the Academy and perhaps the private tuition of at least a few pupils, contributed in a large measure to cause his ill-health. He was at this time preparing a sequel to his series of elementary textbooks and directing his attention to the preparation of a geometry adapted to the needs of the Academy. Restorations of *Euclid* were in general use in the schools at the time. Whiston's edition or Barrow's were the standard texts until Robert Simson published his edition in 1756. Few

of the geometry texts represented any great departure from the original *Euclid*, and Simpson's proposed plan of adapting the subject matter to the capacities of the students, and of changing the original order of the propositions in *Euclid* was a radical one. It must have taken him the best part of the year 1746 previous to his illness to prepare the work.

PUBLICATION OF "THE ELEMENTS OF GEOMETRY"

Simpson's geometry was published in 1747 under the title of *The Elements of Geometry, with their Application to the Mensuration of Superficies and Solids, to the determination of Maxima and Minima, and to the Construction of a great Variety of geometrical Problems*, 8vo. Simpson wrote the textbook for the use of beginners, particularly for the use of the young gentlemen educated at the Royal Military Academy at Woolwich, and he dedicated it to Charles Frederick, Esq., Surveyor General of the Ordnance.

Controversy with Muller in Regard to the Geometry. Simpson's textbook on geometry was on the whole unfortunate because of a charge of plagiarism from Mr. Muller, professor of fortification and artillery at Woolwich. This charge appeared in the closing paragraph of the preface to Muller's *Elements of Mathematics* printed in two volumes in 1748, and reads as follows:

"There had lately been published a book under the title of Elements of Geometry, designed for the Use of schools, which is an incorrect Copy of the first eight Sections of this Work, lent the pretended Author upon a particular Occasion, and printed in a Spurious manner, without my knowledge or consent; an Action too scandalous for any Man of honour to be guilty of. The Editor imagined, I suppose, that the changing some propositions and mangling the demonstrations of others, was a sufficient Disguise to make it pass for his own performance; but how far this will justify such a piece of pyracy, must be left to the judgment of the public."

Simpson made an immediate answer to this charge by placing a copy of the theorems to which Muller refers where others could examine them and form their own conclusions. He made the following statement of his case:

"Those very papers, as they were given out in ye Academy

(which have afforded him a handle to stigmatise me, in requital of my condescension) are now in ye custody of J. Nourse, Bookseller, in ye Strand, where any Gentleman, who has ye Curiosity, may soon satisfye himself as to ye Merits thereof, and how little Reason ye Author had to fall upon me: It will plainly appear that my plan is quite different from his, and that I have not in ye whole, one single proposition or Demonstration that he Can lay any claim to unless he imagines everything to be his own he has raked to-gether from different Authors—That we both have some Demonstrations in common from Euclid, will upon Examination, appear; but they were doubtless as free for me as him; . . . This ye papers themselves which any person has ye Liberty to examine, will sufficiently testify—They are not in my own handwriting (and therefore there can be no pretensions to Forgery) but were copied from a fair, bound, MS. of his, by a Friend of mine, one Mr. Rollinson (clerk to Mr. Hill in Park-Street) who, besides writing a neat Hand, is a good Mathematician and a person of a remarkable good character. . . . "

In the preface to the second edition of the *Geometry* in 1760, Simpson published another reply:

"Were I to attempt to describe the ideas excited in my mind by the singular modesty of this important and solemn appeal to the publick, I should be at loss for fit words to express them without transgressing the bounds of decency.

"But I hope that I have not served so ill of the publick, to be thought capable of acting so very humble a part, as that of copying from *this* author, and of mangling *his* demonstrations, in order to make them pass for *my own*. That a manuscript of his (containing between 20 and 30 of the Principal Theorems in Geometry, extremely ill digested) came into my hands, is indeed true; but it was not *lent me*, but forced upon me, by himself (the very night after my removal to Woolwich) in virtue of an *article* in the original rules and instructions for the Academy; whereby it is ordered, that the second master shall teach geometry under the direction of the first master. But this well intended article, which has been made subservient to the purposes of ignorant tyranny, and daring calumny, has since, in consequence of a publick examination, been annulled by an express order of the Master-General of the Ordnance.

"I could mention some particulars, supported by good authority that occured in the course of that examination, which would ill agree with the importance he assumes in his confident accusations; but I do not think it worth while: This gentleman has, himself, by his different publications, so well convinced the world of his abilities as to render any farther comment on that head unnecessary and ineffectual."

Second Edition of the Geometry and Controversy with Robert Simson. The second, or 1760 edition of Simpson *Elements* was published "with large Alterations and Additions." These changes from the first edition were in part a result of criticisms made by Robert Simson. Dr. Simson, as a restorer of the ancient analysis, did not approve of some of the changes advocated by Thomas Simpson, and in his edition of *Euclid* in 1756 he included critical notes with comments upon many contemporary texts, and upon Simpson's in particular.

Thomas Simpson's textbook most nearly resembled the work of both Clairaut and Legendre on the continent. On the whole his work is clear and precise, and, while representing a radical departure from the Greek original, is desirable. The text is brief, and is well organized as to order and arrangement. There is a possibility that both Robert Simson and Thomas Simpson received some of the ideas presented in their textbooks from the work of Clairaut, for the former mentions Clairaut's treatise in his critical notes, and the latter has produced a textbook more nearly resembling Clairaut's treatise than any other published in England during the eighteenth century.

In speaking of proposition 35 of Book I in his *Euclid,* Robert Simson remarks,

"The demonstration of this proposition [parallelograms on the same base, and between the same parallels are equal], is changed, because there would be three cases to be separately demonstrated, as is done in the translation from the Arabic; for in the Elements no case of a proposition that requires a different demonstration ought to be omitted. On this account we have chosen the method which Mons. Clairault has given, *the first of any,* as far as I know, in his Elements, p. 21, and which afterwards Mr. Simpson gives in his page 32. But whereas Mr. Simpson makes use of Prop. 26, b.l. from which the equality of the two triangles does not

immediately follow, because to prove that, the 4 of b.l. must likewise be made use of, as may be seen in the very same case in the 34 prop. b.l."[18]

Thomas Simpson was the first to object to the reasoning in the twenty-fourth proposition in the first book of Euclid's *Elements*.[19] His objection was ill received by Robert Simson, who discussed the matter at great length in the critical notes accompanying his *Euclid*.[20] Although Dr. Simson's attitude toward Thomas Simpson was mainly critical, it must be said that the treatises of Clairaut and Simpson were the only adaptations of *Euclid* which he troubled to recognize.

Thomas Simpson made several corrections in the second edition of his work for which Robert Simson was no doubt responsible. He also called attention to an admitted error in Dr. Simson's work,[21] but on the whole the controversy was a mild one. Both men were outstanding figures of the eighteenth century, the one a leader in the perfection of subject matter, and the other successful in adapting subject matter to the needs and capacities of the student.

French Translation of the Geometry. Simpson's *Elements of Geometry* was translated into French and enjoyed three such editions during the eighteenth century. The first appeared in 1755, together with two essays, one on *Maxima and Minima* and another on *Problems*. Fortunately a letter written by Simpson to the translator has been preserved:

"Sir,

"Your Letter, by which I think myself very much honour'd, came safe to hand; and it is not without some degree of pleasure that I understand you are engaged in so commendable a design as endeavouring to restore a Taste to Geometry, a Science which of all Others, affords ye strongest Evidence and satisfaction to y^{e}

[18] *Euclidis Elementorum, Libri priores sex, . . .* Glasgow, 1756; Amer. Philadelphia edition, 1810, p. 307.

[19] See *The Elements of Geometry*, by Thomas Simpson, London, 2nd ed., 1760, p. 262.

[20] *Elements of Euclid*, by Robert Simson, Amer. ed., Phila., 1810, p. 302.

[21] *Elements of Geometry*, 2nd ed., 1760, p. 262. The passage to which Simpson refers occurs in Simson's *Elements of Euclid*, Amer. ed., Phila., 1810, p. 348.

Mind; and which as you justly observe hath greatly suffered from ye encroachment of ye modern analysis.

"My little Tract on this Subject which you think worthy of Translation, will admit, I am satisfied of many alterations for ye better; though I flatter myself at ye same time that there is nothing in it very much amiss. If opportunity would permit me to go through these Alterations, I should, without Reserve, communicate them to you; but as my Thoughts are, at present, wholly taken up in another Work (now in ye press), and as your undertaking seems also to be in great Forwardness, I must content myself with transmitting such of them only as I have already by me which I take to be ye most material.

"Concerning the rule at p. 145 which you mention in particular, I agree with you Sir that it will not always be easy for a learner to make out ye Demonstration (without some small Knowledge in Analysis). But, if ye common Rule for ye content of ye Frustrum of a cone (which I have also omitted) had been previously demonstrated, the thing would have been sufficiently easy; since, by taking ye Frustrum generated by *HIDA* from ye Cylinder generated by *HLDA* (vid. p. 128) ye required content of ye segment generated by *AHK*, is given—There is also another Method whereby ye Rule in question may be deduced (independent of that for ye Frustrum of a Cone) but then ye plan of Demonstration to Theor. 6 must be quite altered. Your own judgment will direct you how to act in this particular. Believe me, Sir, that I sincerely wish you Success in your Design, with great respect I am,

"Your most humble and most obedient
"Servant, T.S."

"Woolwich, near London, May 2.O.S. 1752."

"P.S. But if anything amiss should occur to you by me overlooked, I hope you will be so good as to rectify it. Some of ye quotations I know to be wrong, but, these are easily discovered."

Three years after this letter was written the first French[22] edition appeared in print. In the "avertissement du traducteur,"

[22] *Eléméns de Géomètrie*, Traduits de l'Anglois, de M. Thomas Simpson, de la Société Royale de Londres, professeur de Mâthematiques à Woolwich, A Paris De l'Imprimerie de Vincent, rue S. Severin. MDCCLV, avec Approbation, & Privilege du Roi.

the translator gives the same reasons for making the French translation as had influenced Simpson in writing the book. The translator felt that it was rarely that rigor was not sacrificed in contemporary textbooks in a desire to explain things more simply and clearly, but that Simpson had combined the two in a way to make "himself of use to the young people who are destined to study Geometry. . . . "

Of the two essays annexed to the *Elements*, he says in substance, that "this material, which previously had been left to algebraists alone who derived their constructions from analytical solutions, is taken to the door of the beginners. It becomes an extension of the *Elements of Geometry* accustoming the students unconsciously to the consideration of problems still more complicated, and leading to an appreciation of the geometric spirit of the ancients which, should analysis be employed too early in mathematical study, would be lost to the student."

For the most part the changes from the original English edition were suggested by Thomas Simpson himself, but the translator provided more extensive demonstrations in certain cases, notably in the measurement of solids, cones, and segments of spheres.

Two other French editions appeared after Simpson's death, one in 1766 and the other in 1771. The last of these French editions differed from the first two in its "Avertissement" in which it is stated that since the demonstrations of contemporary French texts were so long and tedious, a translation of Simpson's treatise is an absolute necessity. This preface also mentions the fact that many contemporary French texts followed the usual order of Euclid, which is not to everyone's taste. His textbook was well received in France, and the first edition was exhausted almost as soon as it left the press.

Simpson found the publication of his books in other languages pleasing and gratifying, as is evidenced by the fact that in presenting the case to the Master-General of the Ordnance during his trouble with Muller at the Academy, Simpson did not hesitate to mention that his book had been "extremely well received by ye publick and had been translated into different languages."

Influence in the American Colonies. Such books as Simpson's *Geometry* were never as popular in the American Colonies as were

the translations of *Euclid*. When an American reprint of Simpson's *Trigonometry* (1748) was published in 1810 the work had been revised to suit in references the more popular works on geometry by Robert Simson and Playfair, and a frank statement of the reason based on popularity was printed in the preface.

Influence on Later English Writers. Benjamin Donn, author of an *Essay on the Elements of Geometry*, justified a change from the Euclidean treatment of proportion by quoting Thomas Simpson:

"But as some may perhaps expect proof of our assertion that it is too difficult [the doctrine of proportion as given in *Euclid*], it may be proper to observe, that this evidently appears from the great Disputes which have been concerning the Principles on which the Euclidean Method is grounded amongst the great geometricians. . . .

"Mr. Thomas Simpson, one of the best, as well as one of the latest Writers on Geometry, thus expresses himself, 'That there is something very ingenious and subtle in the Doctrine of Proportions, as deliver'd in Euclid's Fifth Book, cannot be denied. All that I contend for is, that the Principles on which it is built are *obscure*, and not so firmly established as to authorize its Partisans to assume that great superiority they lay Claim to, in Point of Geometrical Strictness.' "[23]

Playfair in his *Elements of Geometry*, reviews the controversy between Thomas Simpson and Robert Simson, and he also presents a summary of the various definitions of parallels which Simpson has given. While opposing the definition as given by Simpson in the first edition of the *Geometry*, he heartily sanctions a later one included in the second edition as being "superior in simplicity."

In the first edition Simpson has given the same definition as Wolfius and Boscovitch gave in their texts: that is, assuming perpendiculars drawn from any point whatever in the one line to the other, and "if these perpendiculars be every where the same length, the straight lines are called parallel."

[23] *The Accountant and Geometrician: containing the doctrine of circulating decimals, Logarithms, Book-Keeping and Plane Geometry, etc.*, 4 pt., London, 1765.

Speaking of the definition Simpson has given in the second edition, Playfair says, "that which is given by Mr. Thomas Simpson, in the second edition of his Elements which is very different from that of the first edition where he lays down the faulty definition of parallel lines that was already taken notice of. . . . It is that 'if two points in a straight line are posited at unequal distances from another straight line in the same plane, those two lines being indefinitely produced on the side of the least distance will meet one another'."[24]

PUBLICATION OF A TRIGONOMETRY

In 1748 Simpson brought out a treatise on trigonometry entitled, *Trigonometry, Plane and Spherical, with the Construction and Application to Logarithms*.[25] Simpson wrote the work in the form of an appendix to his *Geometry*, and therefore has given definite page references to that work. He has introduced a notation of simplified formulas by the simple expedient of designating the angles of a triangle by A, B, C, and the opposite sides by a, b, c, respectively. Although in common use today, this was a decided improvement in the practice of his times. He re-introduced the notation simultaneously with Euler, it having once before appeared in a pamphlet by Ri. Rawlinson at Oxford sometime between 1655 and 1668.

The *Trigonometry* also contains excellent proofs[26] of two formulas for plane triangles, $(a + b):c = \cos \frac{1}{2}(A - B):\sin \frac{1}{2} C$ and $(a - b):c = \sin \frac{1}{2}(A - B):\cos \frac{1}{2} C$, which have been ascribed to the German astronomer Karl Brandon Mollweide (1774–1825), who developed them much later. The first formula was given in different notation by Newton in his *Universal Arithmetick;* and both formulas are given by Frederick Wilhelm Oppel in 1746. The trigonometry is extremely brief, consisting of but seventy-nine pages. There is in addition to definitions, a few theorems, corollaries, and propositions of plane trigonometry, a section on the

[24] Playfair, *Elements of Geometry*, Edinburgh, 1795, critical appendix of Notes on the first book of the Elements, Prop. XXIX, pp. 289, 290.

[25] Cantor, *Vorlesungen über Geschichte der Mathematik*, III, 513 f., 514, 515.

[26] H. Wieleitner, *Bibliotheca Mathematica*, 3 S., Vol. 14, pp. 348, 349.

construction of the table of sines, tangents, and secants; to which is added a discussion of spherical trigonometry, a discussion of the nature and construction of logarithms, of the properties of sines, and tangents, of the properties of the plane triangle, and finally of the properties of spherical triangles.

The work was particularly known for the treatment of logarithms. Dr. Halley was the first to show how, by the help of Sir Isaac Newton's binomial theorem, the doctrine of logarithms might be explained very easily using principles of pure arithmetic; he was followed by Sharp and by Simpson. Later writers on trigonometry and logarithms borrowed freely from Simpson, because his method, as one author remarked, "does not require any knowledge of algebra in the reader. . . ."[27]

An American edition of the *Trigonometry* was printed at Philadelphia in 1810 but with alterations. The work was adapted to Playfair's *Geometry* with the additional assurance from the editors that the treatise "will mostly apply to Simson's translation of Euclid's Elements." A second edition appeared in England in 1765, and a French translation by Jombert was published in 1771.

Correspondence on Trigonometrical Subjects. Only one letter containing a discussion of trigonometry is preserved among the Simpsoniana. This communication, dated May 3, 1748, is from John Watts of Comb Abbey near Coventry, with whom Simpson had previously corresponded:

"Sir,

"In a multitude of necessary business I some time find a leisure hour which I joyfull dedicate to the Mathematical Muse; but her lessons being of so severe (tho bewitchingly pleasant) a nature admit there is but here and there a genius worthy her instructions. Therefore who must I appeal to in my distress but to one so much her darling, so eminently distinguished above all as your self; especially as I'm so happy to know there is such a Man as you. Thus to you I humbly appeal, to you I offer my request, that you'll permit me to lay the difficulties I may meet with in the Mathematical Art before you, & that you'll con-

[27] See the preface to *The Elements of Navigation*, by John Robertson, F.R.S., London, 1754.

descend to instruct me, & suffer me now & then to trouble you with a letter. And if it be in my power to gratify you I shall joyfully do it.

"The present difficulties I labour under are owing to an expression in *your Fluxions* and a problem in *Harris's Trigonometry*. The problem in Harris' is in page 150, how to describe the representation of a great Circle passing through two given points. I shall omit transcribing the problem supposing you to have the book by you. He supposes the Line *GH* to pass thro *C* which is the thing to be proved for if it does not the whole is false. By drawing the figure it appears to be true but I want (and cant find out) a demonstration.

"In your Fluxions, page 115, you say The given expression

$$\overline{a + cz^n}\Big|^m \times b\dot{z}z^{m+n-1}$$

may be reduced to this form

$$\overline{az^{\frac{p}{1+m}} + cz^{\frac{p+n+nm}{1+m}}}\Big| \times bz\dot{z}^{\frac{p+n+nm}{1+m}}$$

and it is &c. I wish you would make the whole page a little plainer to me. I hope you or some of your pupils will vindicate your *Ẋns* from the attack Heath has made on them. I should be glad to know how I may get the nos. of the Mathematician which are published and your *Trigonometry* intended to be bound with your *Geometry*.

"I am your humble servant,
"John Watts."

CHAPTER IX

Publication of "The Doctrine and Application of Fluxions"

Renewed controversy with Robert Heath

A renewed controversy with Robert Heath began about the time when Heath became editor of the *Ladies' Diary*. Ostensibly, the point at issue was the interpretation of the doctrine of fluxions, but as a matter of fact Heath was not so concerned over this as anxious for an opportunity to attack Simpson.

In an article which appeared in the *Ladies' Diary* for 1746, Heath very cleverly recommended the *Doctrine of Fluxions* by Emerson, that had been published the previous year, and at the same time he belittled Simpson's method of fluxions without actually naming him or his book in the process. The part of his article which most nearly concerned Simpson reads as follows:

"Those who desire further Satisfaction as to the Nature of Fluxions, of their noble Use and transcendant Excellence, may consult Mr. Emerson's *Doctrine* of the *whole Art*, which is . . . the best of any. . . . Those writers will find themselves mistaken, who pretended to derive the finite Ratios of Motion, or Fluxions producing Magnitudes, without the previous Consideration of Increments, which include the very Notion of what a Fluxion is. This *some* have attempted by multiplying Quantities into their Velocity, and some by other Means, the Result of which originally depends on *incremental Principles*, if they would consider the Matter as far as it will go. . . . " Such criticisms may have been the causes which led Simpson seriously to consider writing a new text on the subject.

Publication of "The Doctrine and Application of Fluxions"

The year 1750 marked the publication of Simpson's *Doctrine and Applications of Fluxions; containing (besides what is common on the Subject), a Number of new Improvements in the Theory, and the Solution of a Variety of new and very interesting Prob-*

lems in different Branches of the Mathematicks,—in 8 vo.[1] The treatise appeared in two volumes and was dedicated to the Right Honorable George Parker, second Earl of Macclesfield,[2] who was then president of the Royal Society. In writing to the Earl of Macclesfield for permission to make the dedication Simpson expressed himself as follows:

"My Lord,

"Whatever Lusture works of Learning may derive from ye patronage of Those in High Stations, it is your Lordships personal Acquirement and extensive knowledge in the Mathematical Sciences that my ambition of desiring leave to prefix your Lordship's Name to this Performance is to be imputed. . . And the world will I Believe do me the justice to allow, that there is no person to whom the Subject of these Sheets can be more properly inscribed than ye Earl of Macclesfield.

"Were your character my Lord less distinguished the obligations I have to your Lordships goodness would alone be motives sufficient to make me gladly embrace this opportunity of publickly expressing my gratitude, and of testifying the Great Esteem and Deference with which I am My Lord, &c."

The dedicatory page of the work on fluxions was modest and unassuming. Simpson states,

"I am not vain enough to imagine that, to one so deeply read in these abstruse and curious speculations, as your Lordship is universally allowed to be, the work will appear without faults but then I have the satisfaction to think, on the other hand, that whatever is here to be met with capable of bearing the test of exact and solid judgment, will also have its due weight, and not fail of receiving your Lordship's approbation: and if, upon the whole, there is merit enough found to entitle me to a favourable reception, it will gratify the highest ambition of, My Lord, Your Lordship's Most Obedient and Humble Servant, Thomas Simpson."

The work appeared as a new edition for reasons best stated by Simpson in the preface, rather than as a second edition of the *Treatise of Fluxions* which was published in 1737:

[1] Cajori, *History of Fluxions*, p. 194.

[2] He was known to have been a pupil of De Moivre.

"Having in the year 1737, published a piece, on this subject, under the title *A Treatise of Fluxions* (whereof the whole impression hath long since been sold) it may be proper here, first of all, to assign reasons why this work is sent abroad into the world as a new book, rather than a second edition of the said treatise. Which in short are these two: First because the present work is vastly more full and comprehensive; secondly, because the principal matters in it, which are also to be met with in that treatise are handled in a different manner."

Simpson acknowledged indebtedness to a "worthy friend"[3] for the method of determining the fluxion of a rectangle, and of illustrating fluxions of higher order. This friend was without a doubt his pupil and patron, Francis Blake, of whom Simpson says by way of acknowledgment, "my obligation I owe to a small Tract, intitled 'An Explanation of Fluxions in a Short Essay on the Theory'; printed for W. Innys, Wrote by a worthy Friend of mine (who was too modest to put his name to that, his first Attempt) whose Manner of determining the Fluxion of a Rectangle, and illustrating the higher Orders of Fluxions I have, in particular follow'd, with little or no Variation." As has been remarked, this was the very essay which grew out of their correspondence together.

Simpson's ideas concerning the first principles of fluxions are not essentially different from those given in his former treatise, although the later treatise differs from the former in the mode of exposition. Simpson was aware that the consideration of time introduced into the general definition would perhaps be disliked by those who would have fluxions be "*meer Velocities.*" The advantage of considering them otherwise, that is, not as velocities themselves, but as the magnitudes they would uniformly generate in a given time, appeared to him to obviate any objection on that head. Simpson defines a fluxion as follows: "The Magnitude by which any Flowing Quantity would be uniformly increased, in a given Portion of Time, with the generating Celerity at any proposed Position, or Instant (was it from thence to continue invariable), is the Fluxion of the said Quantity at that Position, or Instant."

In the preface to the work Simpson remarks, "By taking Fluxions

[3] See above, Chapter IV, pp. 41–95; Cajori, *History of Fluxions*, p. 172.

as *meer Velocities*, the Imagination is confin'd, as it were, to a Point, and without proper Care, is insensibly involv'd in metaphysical Difficulties." According to this new method of expressing the matter, Simpson believed that "less Caution in the Learner is necessary," and that the higher orders of fluxious become more intelligible. "Besides," Simpson concludes, "tho' Sir Isaac Newton defines Fluxions to be the *Velocities of Motions*, yet he hath Recourse to the Increments, or Moments, generated in equal Particles of Time, in order to determine those Velocities; which he afterwards teaches us to expound by finite Magnitudes of other Kinds. Without which (as is already hinted above) we could have but very obscure Ideas of higher Orders of Fluxions: For if Motion in (or at) a Point be so difficult to conceive, that Some have, even, gone so far as to dispute the very Existence of Motion, how much more perplexing must it be to form a Conception, not only of the Velocity of a Motion, but also infinite Changes and Affections of It, in one and the same Point, where all the Orders of Fluxions are to be considered. . . "

Berkeley objected to the use of infinitely small quantities, which objection appears to have influenced Simpson in his conclusions:

"Seeing the Notion of a Fluxion, according to our Manner of defining It, supposes an Uniform Motion, it may, perhaps, seem a matter of difficulty, at first View, how the Fluxions of Quantities, generated by Means of accelerated and retarded Motions, can be rightly assigned; since not any, the least, Time can be taken during which the generating Celerity continues the same: Here, indeed, we cannot express the Fluxion by any Increment or Space, *actually* generated in a given Time (as in uniform Motion). But, then, we can easily determine, what the contemporary Increment, or generated Space *would be*, if the Acceleration, or Retardation, was to cease at the proposed Position in which the Fluxion is to be found: Whence the true Fluxion, itself, will be obtained, without the Assistance of *infinitely small Quantities, or any metaphysical Considerations.*"

Further influence of Berkeley's *Analyst* upon Simpson is evidenced by the following paragraph taken from a letter written by him October 1, 1750, probably to Francis Blake. The letter was drawn up in answer to a criticism of the *Doctrine and Application of Fluxions* to the effect that there were insufficient applications

included of the method of obtaining the fluxion of a rectangle. Simpson explained that the method which is so easily understood is not as easily put to practice without "three times the work required in the common way." In his argument he says, "that the fluxion of a power whether whole or broken, may be derived from ye Fluxion of a Rectangle is well enough known: But to this method I still prefer my own as more direct; in which also ye idea of Velocity (whereon that of a fluxion depends) is better kept up and supported. Neither is it liable to the common objection brought against ye Doctrine of Ultimate Ratios; for as ye quantities to be compared (or ye velocities of m & n) do not vanish at R & S when their ratio is to be taken, there is no necessity of conjuring up *ye ghosts of Departed Quantities to supply their places.*" This last expression is quoted from Bishop Berkeley.

As late as 1823 Simpson's treatise on *The Doctrine and Application of Fluxions* was published in a carefully revised edition, and adapted by copious appendices, to "the present advanced state of science," by a graduate of the University of Cambridge. An "advertisement" included in the front of this later edition contains the following remarks:

"An author of Simpson's celebrity needs no recommendation. The ablest Geometers of this and other countries have at all times bestowed upon him the most liberal praise. The following extracts will serve as specimens:

"Frisius, Vol. I, page 287,

"Demonstrationes olim contrahere studii ut auditorum meorum comodo consulerem et Theoriam Fluxionum paucis exhibere: quod fere eodem in Anglia praestitit Thomas Simpson.

"Woodhouse, *Physical Astronomy*, page 202.

"Thomas Simpson, the ablest Analyst (if we regard the useful purposes of Analytical Sciences) that this Country can boast of, &c.,

"Playfair's *Collected Works*, Vol. 4, page 323.

"At the moment when we now write, the treatises of Maclaurin and Simpson are the best which we have on the Fluxionary Calculus, &c. . . . "

REPUTATION IN THE AMERICAN COLONIES

Simpson's reputation in the American Colonies was very highly regarded, as is shown by a quotation from the eulogy of John

Pickering, a student at Harvard in 1797 who greatly enjoyed the subject of fluxions:

"Near the close of his senior year he received the honor of a mathematical part, which appeared to give him more pleasure than all his other college honors. It afforded him an opportunity to manifest his profound scholarship in a manner most agreeable to his feelings. When he had delivered the corporation and overseers this part, containing solutions of problems by fluxions, he had the rare satisfaction to be told that one of them was more elegant than the solution of the great Simpson who wrote a treatise on fluxions, in which the same problem was solved by him."[4]

CONTROVERSIES CONCERNING THE TREATISE

The Doctrine and Application of Fluxions, in spite of later praise, was not received without some adverse criticism, for, owing to the confusion in the minds of mathematicians as to the correct definition of fluxion, Simpson's conception would hardly meet with everyone's approval.

Consequently, in place of a regular review, *The Monthly Review* published a critical letter which had been submitted to them. The letter, written by James Jurin, was published under the pseudonym of Cantabrigiensis.[5] Jurin's letter occasioned so much controversy for the next two years that it would be well to quote it in full:

"To the Authors of the Review.

"Gentlemen,

"As a methodical *criticism* of Mr. Simpson's late doctrine of fluxions would take up as much room as might suffice for a *competent* treatise on the subject, I shall only offer you a *few* general obscurities thereon, from which some idea of that per-

[4] Cajori, *Teaching and History of Mathematics in the United States*, p. 59.

[5] See *The Monthly Review*, London, December, 1750, v. 4, p. 129, Article XXV. The letter was prefaced by remarks of the editors of the *Review* to the effect that, "The following letter was sent to us by a gentleman, who is undoubtedly master of the subject it relates to; and therefore we chuse to oblige him by inserting it; without however making ourselves answerable for anything it contains. . . . "

formance and likewise of the writer's abilities, with regard to *method, conciseness, perspecuity, accuracy,* and *judgment,* may be formed.

"The largeness of the work, and the confusion with which many things are therein treated, require more *time,* and *patience* to consider than even *judges* can afford to lay out; but to the *learner* are insuperable difficulties to his improvement.

"I find, that these volumes contain the substance of *Mr. Simpson's first book of fluxions,* and his mathematical essays; to which he has added several *new* problems, and some things are copied out of late writers; but I can find little or nothing *new* in them besides.

"The *definition* he gives of fluxions is very odd. He says, it is the "magnitude by which any flowing quantity *would be* uniformly increased."—From which I make these observations: that in quantities *uniformly* generated, the *fluxion* must be the *fluent itself,* or else a part of it. And in other quantities generated by any variable law, the fluxion, according to *him,* will not be *real,* but only an imaginary thing. Likewise his definition (*in which he pretends to exclude velocity*) cannot be made intelligible without introducing velocity into it. Again, he mistakes the *effect* for the *cause;* for the thing generated must owe its existence to something, and this can only be the velocity of its motion; but it can never be the *cause of itself,* as his definition would erroneously suggest; and by the way, one would wonder how it comes to pass, that after all the elaborate performances written on this head, our author should still stick in the *mud,* and run himself into the old exploded method used by foreigners; and which is subject to all the *cavils* that have ever been raised against that science, and which he must necessarily incur, while he is endeavouring to avoid them.

"As to what he says against *Sir Isaac Newton's* using the word *velocity,* 'tis mere idle stuff, and deserves no answer; and only proves, that the author understood nothing at all of what *Sir Isaac* was writing about.

"And as to what he says, in respect to the *goodness* of the method he has taken, and the *improper* method of some authors, he must leave other people to judge of that. But it seems evidently more natural to lay down *principles* first, and then deduce

the *practice*, than to run backwards and forwards in the book, from *one* to the *other*, without any order, or method. And it must be exceedingly troublesome to *hunt* up and down, in a work of 5 or 6 hundred pages, to find a rule, when placed at *random*.

"He farther says, that there are numbers of *forms of fluxions* laid down by authors, which he dares pronounce, "can never arise in any inquiry into nature:" but this is what he does not know, and therefore is a piece of *vanity* and *immodesty* in him; on the contrary, there are *innumerable* forms of fluxions, arising by inquiries into nature, that neither he, nor any one else has laid down. As to his assuming *that title*,[6] I suppose it was due to give his book a character, and to make it sell.

"In *section* 1 *of part* II he lays down the proportions of the fluxions of the sides and angles of spherical triangles; but meddles not with plain ones. This is all taken from Mr. Cotes' *Estimatio errorum*, where it is elegantly demonstrated.

"In p. 293, this fluxionary equation is given,

$$c\,x^2\dot{x} + y\,\dot{x} = a\dot{y};$$

and the *fluent* is found

$$y = -cxx^2 + 2ax + 2ad + dM^{\frac{x}{a}}$$

Here the author says $dM^{\frac{x}{a}}$ serves to correct the fluent, which cannot, without it, be corrected in the usual way, so as to give $x = 0$, when $y = 0$. But if the fluxion of this quantity, without the *exponential*, produces the fluxional equation; then, certainly, this quantity is not necessary. For there are infinite numbers of cases wherein x cannot be $= 0$, when $y = 0$; but the result will be according as the nature of the problem directs.

"In the *9th section* there are several problems that are very *obscure* and *intricate;* particularly in p. 455. How comes $x = \frac{f}{1+g}$, and from thence the fluxion to be transformed into what he gives us?

"In *section* 11, are several *problems* which don't at all belong to *fluxions*, and the like in several other parts of the book. I am

"Sirs, your most humble Servt.

"Cantabrigiensis."

"November 5, 1750."

[6] "*Doctrine and Application of Fluxions*, being Mr. Emerson's title to a treatise of his on this *Subject*."

While the editors of *The Monthly Review* stated that they would welcome any answer that Simpson or his friends saw fit to make, no advantage seems to have been taken of this opportunity. The debate was transferred to the *Ladies' Diary* and to certain new publications that were begun for the express purpose of presenting arguments in favor of Simpson. Friends of Simpson and friends of Emerson appeared in active controversy, while neither of the principals seems to have taken part. It seems natural to suppose in the light of earlier remarks made by Robert Heath in the *Ladies' Diary*, that the controversy for the next few years is due to him rather than to Emerson. In addition, Simpson had appropriated the title of Emerson's work,[7] which, if we are to judge from Jurin's remarks in *The Monthly Review*[8] may have caused ill feeling.

At any rate the publication of the *Doctrine and Application of Fluxions* was the beginning of a series of slanderous articles levelled against Simpson. Most of these articles appeared in the *Ladies' Diary*. John Turner, one of Simpson's friends and students, began a publication known as *Mathematical Exercises*, in order to vindicate him. This journal was popularly known as *Turner's Mathematical Exercises*, and it seems to have been published only so long as the controversy continued. Six numbers[9] in all appeared between 1750 and 1752. Simpson, according to Hutton's account, was "commonly supposed . . . to be the real editor of, or had a principal share in, . . . Turner's *Mathematical Exercises*." There can be no doubt that he entered into his own defense under Turner's name, for articles which appeared in print as Turner's are to be found among Simpson's manuscript material in semi-prepared state in his own handwriting. This is true of an article which defends Simpson against the charge of "mistaking the Effect for the Cause."

An article signed "Honestus" appeared in the *Daily Gazetter* for December 4, 1750, which accuses Heath of reserving the best

[7] See Cajori, *History of Fluxions*, p. 192.

[8] See above, p. 157 footnote.

[9] Readers were invited to send, "their Performances (whether new Problems, Paradoxes, Solutions, etc.) Post paid, to be left with Mr. James Morgan, at the Three-Cranes, in Thames-Street. . . . "

material of the *Diary* for his privately owned publication the *Palladium.* "Honestus" is supposedly Turner. Heath answers this charge in the same chronicle.

The controversy reached its climax in a publication termed by Turner "a scurrilous Pamphlet." This pamphlet was entitled *Truth Triumphant:* or *Fluxions for the Ladies,*[10] and was published at London, in 1752. As the pamphlet is made up of different articles, it is possible that prior to this all of them had been published separately. The first pamphlet in *Truth Triumphant* is a clever play on the title page and also on Simpson's definition of fluxions, and its alleged defects. The style of the articles appearing under the pseudonyms of "$\dot{X}$ Primus" and "$\dot{Y}$ Secundus" indicates that they were the work of Robert Heath, assisted by adherents of William Emerson. These articles by "$\dot{X}$ Primus" and "$\dot{Y}$ Secundus" are written as criticisms of two essays by John Turner in defense of Simpson. In the article signed "$\dot{X}$ Primus," Turner is treated with much contempt:

"Who this John Turner is, whether he is Mr. Simpson's Clerk, or his Pupil, or some Dependent on him; or whether he be Mr. Simpson himself, is not very material to the Reader. . . ." The second reply accuses Turner of being "equally in the Dark" with Simpson.

The most interesting part of the whole pamphlet is entitled "Animadversions on Mr. Simpson's Fluxions" "By $\dot{Z}$ Tertius," quoting a criticism of Simpson from the pen of John Landen. Where Landen's review first appeared is not determined. As quoted here, Landen objects to Simpson's definition of fluxions "as *faulty,* by the Author's different *Idea* given of them to that by the Inventor." Landen is represented as disapproving of "denoting all Quantities whatsoever by Lines, to bring them to one Denomination, and those Lines, to be described by Bodies in Motion."

Landen criticises fluxions in general and remarks that "the

[10] *Truth Triumphant: or, Fluxions for the Ladies. Shewing the Cause to be before the Effect, and different from it; That Space is not Speed, nor Magnitude Motion. With a Philosophic Vision, Most humbly dedicated to his Illustrious High and Serene Excellence, the Sun. For the Information of the Public, by $\dot{X}$, $\dot{Y}$, and $\dot{Z}$ Who Are Not of the Family of x, y, and z, but near Relations of $\acute{X}$, $\acute{Y}$, and $\acute{Z}$* . . . London. Printed for W. Owen, MDCCLII.

finding, from the velocities, the spaces passed over, and *vice versa*, may be managed by common *Algebra*, without the least Obscurity. The Business had always been better considered in that Light, without ever making Use of the Term *Fluxions*, as if a new Kind of Analysis, tho', in *Fact*, only the *Doctrine of Motion* improved, and applied to the Purposes before unthought of."

It was fully four years later that Simpson appears to have heard for the first time from John Rowning[11] that Landen was reported to believe "there was nothing new" in his *Doctrine of Fluxions*. Simpson then wrote Landen in 1758 with whom he was on friendly terms as if surprised that he could be so quoted.

The next article in *Truth Triumphant* is a reprint of Jurin's article which was first published in *The Monthly Review*. Eight more articles concerning motion, fluxions, and mechanics bring the whole pamphlet to a close.

The controversy was continued in Turner's *Mathematical Exercises*, number five being an answer to articles appearing during the years 1751 and 1752 in the *Ladies' Diary*, in the *Lady's Philosopher*, and in a new magazine, the *Palladium*.[12]

The *Ladies' Diary* for the year 1751 contains the statement, "And tho' Mr. *Simpson* has pretended to deduce the *Ratios* of Fluxions of Quantities without the use of *Indefinitely small Quantities* (see his new *Doctrine and Application of Fluxions*) yet the Motion of his *Points* along the *Lines* answers to them by the indefinitely small Spaces described together, and are to the same Effect as Quantities taken indefinitely small; which Sir Isaac Newton himself introduced to illustrate the Quantity of relative motion by. *Fluxions*, as instantaneous Velocities, are also as the

[11] Author of *A Preliminary Discourse to an intended Treatise on the Fluxionary Method*, London, 1756. See Cajori, *History of Fluxions*, p. 198.

[12] De Morgan, *Arithmetical Books*, London, 1847, page 72, contains a note in reference to this publication. De Morgan says, "in 1749 Heath had commenced the *Palladium*, an annual publication resembling the *Ladies' Diary*, the very first mathematical question of which is so expressed as to convey an indecent double-meaning, in a manner obviously intended. From 1750 or thereabouts, he began to write against Thomas Simpson. One of his publications against the latter is headed, 'Miss Billingsgate in a salivation for a blackeye;' and in a letter published in a newspaper, in 1751, he remarks of Simpson and another, that 'The best writer against both is one who shall sign the warrant for their execution.' "

Increases or Quantities of Space passed over together by those *instantaneous Velocities*, uniformly continued; but are not the Spaces or Quantities themselves that *would be* described by them according to Mr. *Simpson's* new Theory, and Application. See *Emerson's* Doctrine and Application of Fluxions. Price 6s. only." The *Ladies' Diary* for 1752 printed a more satirical continuation of this article.

The controversy was carried on by the rank and file of mathematicians and never rose above the level of mere bickering. *Truth Triumphant*[13] summarized the animadversions made against Simpson during the period of two years 1750–52. It was not, however, until 1754 when Simpson himself succeeded in becoming editor of the *Ladies' Diary* that the dispute frankly closed.

[13] See *The Monthly Review*, London, 1751, vol. V, p. 462, where *Truth Triumphant* is described as "an odd assemblage of controversial scraps, chiefly relating to some disputes concerning Mr. Emerson's treatise on fluctions, and Mr. Simpson's on the same subject. This most unimportant controversy was first occasioned by the few observations on Mr. Simpson's book published in *The Monthly Review*. . . . The author writes in a manner that can do little honour to any party or opinion. And why he chose to give this strange and insignificant production so odd a title, is a mystery that none but himself can clear up."

CHAPTER X

Later Publications

Publication of "Select Exercises for Young Proficients in the Mathematicks"

Meanwhile Simpson redirected his attention to publishing subject matter of immediate use in the Academy or similar institutions. He therefore brought out in 1752 a treatise entitled *Select Exercises for Young Proficients in the Mathematicks.* The greatest part of this work was originally prepared for classes in the Academy, but as it was Simpson's belief that it would be generally useful, especially to those in public teaching, he prepared it for the press. The work itself consists of six parts, or tracts.

1. The first part contains a number of algebraical problems, and their solutions, of such simplicity as to fit the needs of beginners. In addition to these problems and their solutions, a great number of which are entirely original, Simpson has also included work with equations and illustrated various methods of substitution.

2. The second part consists of geometrical problems with algebraic and geometric solutions. "In this part," writes Simpson, "the learner will find a large field to exercise his industry in: he will moreover have the opportunity to compare the two methods together, and from thence observing that sometimes the one has advantage and sometimes the other; that in some cases they both proceed upon the very same properties, and in others upon quite different ones: and it may be further remarked from hence (which will be of some use to know) that, when quantities are given in magnitude only, the algebraical method generally claims the preference, in point of case and expedition at least; whereas the advantage is almost always on the side of the geometrical effection, when the position of points and lines, and the quantities of angles are given."

On one or two particulars this tract is open to criticism. The first is the use of algebraic symbolism, which was considered by

many of his day to detract from the rigor and strictness of geometry. "But," says Simpson, "it is not the use of symbols (which some more scrupulous than discerning, have condemned) but the ideas annexed to them that render the consideration geometrical, or ungeometrical. In pure geometry regard is always had to the absolute quantity of some one of the three kinds of extension, abstractedly considered; and whatever symbols are used here. They are to be considered as expressive of the quantities themselves, and not as any measures, or numerical values of them. Thus $A \times B$, taken in a geometrical sense, we have an idea, not of the product of two numbers (as in the algebraic notation), but of a real, rectangular, space comprehended under two right-lines, represented by A and B, and two others equal to them. So, likewise, $\frac{B \times C}{A}$ is not to be understood here in the light of an algebraic fraction, but as a right-line, which is the fourth proportional to three other right-lines, represented by A, B, and C. These distinctions are absolutely necessary to those who would have an accurate idea of the subject."

On a second ground Simpson believed he might meet with some criticism because he referred students to his own *Elements of Geometry* rather than to Euclid. Simpson thought that he was justified in so doing because the students who were to use this as a textbook were prepared in the first principles of geometry from *Elements* other than those of Euclid. Besides he had included in his *Elements* a number of new propositions found only in his or other modern texts.

3. The third tract included in the *Select Exercises*, is a Theory of Gunnery independent of conic sections. Besides the motion of projectiles, "practical applications with their solutions" depending on the theory are given by plane trigonometry, "as well those where the object is elevated or depressed as where it is situated in the plane of the horizon."

4. The fourth part is a consideration of "a new and comprehensive method of finding the roots of equations in numbers," without the necessity of "repeating the operation, as in the common way, by converging series."

5. The fifth tract is an account of the nature of fluxions with an investigation of the fundamental rules. This tract Simpson

thought would be of use to beginners and also to those who though familiar with "the practice and application of Fluxions, have nevertheless but an imperfect Idea of the first Principles of this difficult branch of Science." The same definitions and explanations of the fundamentals are given by Simpson as occur in the 1750 edition of his *Fluxions*.

6. The last part was supposed to supplement his *Doctrine of Annuities and Reversions*, and gives a valuation of annuities for single and joint lives, with a set of new tasks, but as Simpson thought it was too small to publish separately, he had it inserted at the end of this work. It was, however, removed from here in later editions and inserted at the end of the *Annuities* where it really belongs. The examples and tables given in this last tract are calculated according to the London bills of mortality; but the solutions are general so that applications can be made to any other table of observations.

Simpson dedicated this volume to John Bacon Esq., F.R.S., of Newtoncap, who, if one can judge from the dedication was a gentleman with an appreciative knowledge of mathematics. Such collections as the *Select Exercises for Young Proficients* were very popular during the eighteenth century.[1]

CORRESPONDENCE CONCERNING ANNUITIES AND REVERSIONS

The additional essay, or supplement, discussing annuities and reversions included in the *Select Exercises for Young Proficients*, was no doubt inspired in part by communications which Simpson had received following the publication of the original treatise. Many people appealed to him for practical applications to their own problems. For instance, Edward Dickinson of Carey Street, London, first wrote theoretically[2] of a problem which he later admitted dealt with his own estate. He speaks of it in the following manner:

"It is indeed my own estate, & there being some Question between my lessor & me about the serving of renewall: I am desirous of knowing what in truth I ought to pay for it & therefore I have stated to you the whole Income & Disbursments of

[1] *The Monthly Review*, July, 1752, volume 7, page 78, no. XV.

[2] Letter dated Carey Street, 14, December, 1750.

the last 7 years with exactness, & I must observe to you that no extraordinary Damage or Accidents have happened during that time to enhance the Repairs.

"It is to be noted that the Houses being built soon after the fire of London, have now stood upwards of 80 years & At the Expiration of the 14 yrs. I have to come in, ye Lease will have stood within 2 or 3 years of 100. So that what I am now purchasing is a reversion of these Houses 14 yrs hence for 7 years. I believe you will not imagine the repairs will then be less than they were the last 7 years, although no great or extraordinary dilapidations should happen in those 7 years. And I am afraid all the other Disbursements will rather Increase than Decrease upon my hands during that time.

"I shall be much Obliged to you for your answer at your first Convenience, because we are now in Treaty, & I have promised to give my final Resolution, which will be founded upon your Opinion. I desire your thots upon this affair as also the accounts may be sealed up. . . .

"I gave your Name & address to Sir Lionel Pilkington who had a matter of Calculation to advise upon."[3]

Many such questions were sent to Simpson for answer, so as to leave no doubt of the fact that this type of problem was of great popular interest. Simpson's ability to answer such questions was based upon as he says "ye Result of Calculations founded on a great number of Observations on ye Bills of Mortality of different places."

LATER COMMUNICATIONS FROM WILLIAM CHAPPLE

As late as October 17, 1760, the year before his death, William Chapple of Powderham Castle near Exeter, wrote asking him for information on this same subject. Chapple's interest was practical in that "as Sir William Courtenay's Steward and otherwise," he was obliged to bargain for leases and reversions granted for lives, which sort of tenure was most frequently in use in his part of the country.

In speaking of the texts on annuities which Simpson had published Chapple says,

[3] Letter written 9, May, 1751, Carey Street to Mr. Thos. Simpson at The Royal Academy at Woolwich, Kent.

"I have in a few leisure hours I could spare from Business, taken some pains in finding a nearer approximation to the values of single lives than the theorem you give in P. 16 on Annuities & P. 315 & 316 of the Mathematical Exercises which theorem I have also found by a different process and thought I might claim it as my own 'till a few year's since when I happened to compare its result with yours; and since find that *Mr. Dodson* has done the like, if he did not borrow it from you at first. . . . I have gone a good way toward collecting materials from Parish registers here in this County, for a Table of Observations, from which I would calculate tables for my own use, the People here being in general longer-lived than in London; but I have not yet finished it. . . . "

SIMPSON'S IDEAS WITH RESPECT TO DODSON

Other great authorities on annuities and reversions who were in London at this time, were De Moivre and Dodson. Dodson had published the first part of *The Mathematical Repository* in 1747. The work was a collection of upwards of 500 questions selected from Oughtred, Leybourn, Moore, Kersey, Wallis, Harriot, Jones, De Moivre, Ward, Parsons, Ronayne, and Simpson of England, besides the leading men of other nations, such as Descartes, Ozanam, the Bernoullis, Wolf, and Euler.

The second volume of the Repository appeared in 1753, and was for the most part devoted to the subjects of insurance and annuities. In the preface Dodson writes,

"Although the valuations of annuities on lives, the subject principally treated of in this book, is of great importance in most parts of the known world and particularly in the British Empire, yet it is a branch of mathematical learning which has been but lately cultivated, and probably not yet brought to that perfection which it is capable of. I have ventured to publish the result of my endeavours to facilitate this kind of interesting and difficult computations, because I conceived I had made some improvement therein. . . . " Dodson seems to have had clear ideas of the worth of De Moivre. In the beginning of this volume is printed the *General method of solving indeterminate questions* written by Abraham de Moivre in the form of a letter to William Jones. It was to have been printed by Jones in the *Philosophical Transactions;* but he died before De Moivre could communicate it to him and consequently it was submitted to Dodson for publica-

tion. Dodson remarked that although he himself had treated the subject comprehensively in the second volume of the *Repository*, De Moivre's treatment was "a more general and elegant handling of the subject" and therefore deserved a place at the beginning of the volume.

A letter written by Simpson April 4, 1755, gives his estimate of this work, which is of interest in the light of the previous disagreement in 1742 between De Moivre and Simpson over the publication of the *Doctrine of Annuities and Reversions:*

"Dear Sir,

"Conformable to your Desire, I take this Opportunity to send you my observations of ye mistakes in Mr. Dodson's Book wherein I have expressed myself without reserve, as I believe you are sensible that I have no design to hurt ye author either in your esteem, or that of ye World; and that (for reasons well known to you) those mistakes will be no farther noticed by me, unless I should find myself under the disagreeable necessity of doing it, in order to clear up Misrepresentations. Tho' (as things are now circumstanced) I reely wish that he had not (by his Treatment of me) given himself room, to look upon me in ye Light of an Enemy.

"The chief of ye said mistakes, which is productive of most unlucky consequences I mention'd to you, is ye principle or proportion laid down in his 19th problem; relating to ye probability of two persons of given ages, being both extinct in a given space of time, under ye restriction of ye younger's surviving ye Elder, or ye Elder's surviving ye Younger. This proportion, you will perceive, he makes to be as m to n (m being ye Complement of ye younger life (A) and n that of y^e Elder (B): But the true proportion, or Ratio is that *of Equality:* Because y^e probability of y^e proposed Events' being determined, or taking place, during any particle, or Interval of Time (t), according to y^e Order of Survivorship A, B, depends, not only on y^e probability) $\left(\frac{t}{n}\right)$ of B's becoming extinct during that Interval (which, alone, is considered by this Author) but likewise on y^e probability $\left(\frac{T}{m}\right)$ that A dies in y^e Time (T) preceding in y^e said Interval: So that $\frac{t}{n} \times \frac{T}{m}$, y^e compound of these two must be y^e true measure of y^e

probability in question. And, for y^e^ very same reason, y^e^ probability on y^e^ said Interval t, according to y^e^ other order of survivorship B, A, will be measured by $\frac{t}{m} \times \frac{T}{n}$, which value and y^e^ former are manifestly equal each to each (for as long as y^e^ two lives can possibly exist to-gether). And consequently y^e^ two required probabilities are in y^e^ Ratio of Equality; and not as m to n. The ill effect of this erroneous principle first appears in y^e^ solution of his 30th problem which, thereby, in many very possible cases differs from ye Truth by more than half ye whole value. Thus, for example, let y^e^ value of y^e^ Reversion mention'd in y^e^ first paragraph of y^e^ said solution, be 1000£, and suppose y^e^ given ages of A and B to be 24 and 58 years, respectively (the life C in Expectation being considered as y^e^ oldest of y^e^ three, according to our author's first Case). Then m being $= 62$, and $n = 18$, the value of y^e^ Expectancy $\left(\frac{n}{n+m} \times 1000\right)$ according to y^e^ order of survivorship A, B, will (by his Rule) come out 225£ only; whereas y^e^ true value is 500£ exactly, as is proved in my *Select Exercises;* where you will find all cases of this problem, resolved.

"The same principle or proportion is again made use of, almost in every line, i.e. from p. 207 to p. 257; whereby, all that is said about ye probabilities of ye different orders of Survivorship of four unequal Lives, is rendered erroneous. But this being a matter of speculation rather than of real use, the oversights therein cannot, I apprehend, be attended with any very ill consequences. Other faults might be pointed out (tho' perhaps none so considerable) but as I have no inclination to take upon ye province of a Critic (notwithstanding ye Author's having assumed so much, and appropriated to himself ye method of solving these Kinds of problems independent of Fluxions, tho' published before in my *Exercises*) I shall conclude here, myself,

"Who am, Dear Sir,
"Your
"most obedient
"Humble Servant,
"T. Simpson."

"Apr. 4, 1755."

As a matter of fact, Simpson's real grievance was due to his belief that Dodson had not given him credit enough for contributions in this field. Among his private notes is an article apparently drawn up for the purpose of establishing charges of plagiarism against Dodson.

For instance, he writes:

"The 2^{d} place I am convinced and it must appear notorious to ye World that ye list of ye general Improvements in ye 12 pages of his Book (from Question 104) are taken from my Select Exercises. In order to satisfy yourself in this please to observe ye last paragraph in p. 22, and the two first of p. 23, in his preface; his 104 Question, the Table in p. 339 and what follows thereof; and ye first part of his solution of his 105 Question. At ye same time please compare these passages with ye last paragraph in p. 313 of my Exercises, the paragraph ending p. 311, and beginning 312; and ye Table at p. 255. I am sensible by such a comparison you will be convinced more so as he seems particularly to desire to appropriate these Improvements as his own, else why did he not when he was telling ye World what Mr. De Moiver had done take notice of another person whose Thoughts upon ye Subject he appears to have been unacquainted with. . . . "

Whether this article was ever published I have been unable to determine, but Simpson had some cause for his comments. Dodson, in his enthusiasm for the work of De Moivre had neglected to take proper recognition of the work of Simpson and others, and probably did assume credit for much that Simpson himself had accomplished in this field.

CHAPTER XI

Editorship of the "Ladies' Diary"

Simpson as editor of the Ladies' Diary

Upon the death of Henry Beighton the editorship of the *Ladies' Diary* passed into the hands of Robert Heath. It is of significance that from the year 1744 when Heath became editor, Thomas Simpson ceased contributing to the magazine. Heath, however, made use of the publication to open numerous controversies, in which as has been mentioned before, Simpson's work on fluxions was subjected to severe criticism. As Simpson observed, the debate chiefly "turned upon ye three following points,

"1. The Author's definition of a Fluxion.

"2. His Method of resolving a Fluxionary Equation.

"3. The assertion of this opponent that *ye Author "publishes nothing but by ye Assistance of Others."*

Beginning with the year 1752, Heath's comments with regard to other men besides Simpson became more satirical, until finally it was thought necessary to take the publication from his hands.[1] Thomas Simpson was appointed editor and compiler of the *Ladies' Diary* beginning with the year 1754, an office which he held for six years. During that period Simpson was also an active contributor although under fictitious names. Just how much influence, if any, he had exerted to have Heath removed from office is difficult to determine.

There was a decided improvement in the composition of the *Diary* under Simpson's administration, including both the mathematical and the general parts. The mathematical part was of course much improved in accuracy. With respect to the general part being of more interest to ladies, Simpson was fortunate in having the assistance of Mrs. Beighton for a time. Many papers

[1] De Morgan, *Arithmetical Books*, p. 75, says, "He was editor of the *Ladies' Diary* from 1745 to 1753, when the Stationers' Company found it absolutely necessary to strike out some of his scurrility, and dismiss him; . . . "

containing attempts of his own in this field are to be found among his private notes. These clearly indicate that however successful Simpson was as a mathematician, he was a failure as a rhymster or expositor on mundane things, or as an advisor similar to our modern "Mrs. Fairfax."

In compiling a work of this sort Simpson was called upon to consider trivialities as of much importance. This was true even in respect to mathematical matters, for most of the prize questions had to meet the low level of ability of the average subscriber, and consequently the problem material was not of a difficult nature. Yet occasionally some interesting material was published, and sometimes was used in college examinations. The following are fair examples during his editorship of the type of material of most interest:

"*Question 401 by Mr. E. Rollinson:*—To determine ye best and most ready method for discovering or nearly approximating ye value of an annuity upon ye longest of any Number of assigned Lives, according to any Table of Observations on ye probability or Degrees of mortality of mankind, by dividing the whole extent of life into different periods, during which the decrements, or numbers dying off yearly, may be esteemed equal; without having any other series to sum than a common geometrical progression."[2]

"*Question 398, by Mr. Richard Gibbons, Ladies' Diary, 1755.*—Being on a journey, I took a guide at Modbury for Dartmouth with whom having traveled 66 minutes, I asked him how far we were come? who replied, Just half so far as we are now from Totness. Having jogged on together seven miles farther, I asked him how far we now had to travel? whose answer was still the same, Just half as far as we are from Totness: These indirect answers I did not like, tho' I found when we arrived at Dartmouth, which we reached in 55 minutes more, that all the fellow had said was strictly true, and that the two roads leading from Totness to Dartmouth and Modbury formed a right angle From hence I would know the true distance of the three towns, and the rate at which we travelled, which was uniform?"

"*Question 411, by Mr. J. Ash, Ladies' Diary, 1756.*—Being on a journey, in lat. 52° 30′ N. the third of April, 1755, in the after-

[2] This was suggested as a prize question during one year of Simpson's editorship.

noon, I observed the two intersections of the inner part of the interior rain-bow with the horizon, to form an angle, at the place where I stood, of 60°: From which the time of observation is required."

"*Question 416, by Mr. Tho. Moss, Ladies' Diary, 1756:*—Suppose 12 half-pence to be thrown up, and those that come up heads to be taken away, and the remaining ones to be thrown up again, and so on, in the same manner, 'till all the half-pence have been thrown up heads: 'Tis proposed to find in what number of throws, according to an equality of chances, this may be effected."

"*Question 433, by Mr. R. Weston, Discip. Landenii, Ladies' Diary, 1757:*—If a straight, uniform, slender rod, or bar, of heavy metal of given length, be left to descend after being set leaning, in a given position, with its lower end (n) on the immoveable horizontal plane AB, and its upper end (m) full against the immoveable vertical plane AC (the lower end being at liberty to slide freely along the first mentioned plane, while the upper end is descending), what will be the position of the rod when it shall cease to touch the said vertical plane? how long will it then have been in motion? and how far from the point A will the end (m) strike the horizontal plane?"

"*Question by Mr. O'Cavannah, of Dublin,* (Thomas Simpson), *Prize Question for the year 1757, Ladies' Diary:*—

"A pert young exciseman, who boasted his knowledge
In guaging of vessels, and taking an ullage,
A wager would lay, his skill to make good;
And the case we proposed for his trial thus stood:
'Eighteen inches, five tenths, a cask's length is given:
'The heads, which are equal, are each thirty-seven.
(Each head diam. is 37 inches).
'It is likewise known, that such is the make,
'The cask it is formed the least wood to take
(The superficies is the least possible).
To find the content quite baffled his art;
But he hopes you the method next year will impart."

CHANGE IN CONTRIBUTORS TO THE DIARY

In a letter to Mrs. Beighton March 19, 1754, Simpson describes the conditions existing with respect to the *Diary*, especially with

regard to the change in contributors. The letter shows clearly the many things Simpson found it necessary to consider in making the compilation each year.

"Madam,

"Your packet from the Stationer's Hall I have received and shall always pay a particular regard to what comes thro' your hands from the old Contributors: But cou'd have wished that you had also sent (as you did last year) the calculations of ye Lunations and ye Moon's Rising and Setting, as I shall not have sufficient leisure (I fear) to make these calculations myself.

"I hope Madam, it will not be displeasing to you to hear that ye Diary, this Year, has met with very great encouragement, near 100 letters having been already received, and many of them from Masterly hands: but, (among a great Number) very few Names of old Contributors appear. Your endeavours to re-engage these Last (by obtaining a promise from some of them, as you inform us) will have a proper effect at Stationer's Hall: The only concern we are under is, that (abounding in good Things) we shall not be able to find so much Room as these Gentlemen may have reason to expect from such an Invitation: However, you may depend upon this, that if we have any partiality it is in favour of ye Old Contributors, and that we shall take a particular pleasure in obliging those whom you reckon in ye Number of your Friends.

A line from you in Relation to ye above mention'd Calculations will be esteemed a Favour, by,

"Madam,
"your most
"Humble Servant,
"The Compiler.

"I am sorry that Mr. Cowper was not taken proper care of in ye last Diary; but it is intirely owing to ye carelessness of ye person who transcribed ye copy for the press."

The fact that Simpson was now compiler of the *Diary* caused a great many of his friends and students to contribute to it, so that this fact in itself brought about a change in contributors. Henry Watson, Thomas Moss, E. Rollinson, W. Bird, Hugh Brown, Christopher Mason, Wm. Kingston, Mr. Robert Shirtcliffe

(author of *Theory and Practice of Gauging*), and J. Landen were among the most prominent contributors.

FURTHER TROUBLE WITH HEATH AS TO THE DIARY

Mrs. Beighton evidently thought that Heath had been ill-treated, and was in communication with him, for as a result, a letter written by Simpson to her appeared in the *Ladies' Chronologer* and *Palladium*. Simpson thereupon felt it his duty to review the cause of Heath's dismissal and remind Mrs. Beighton of her obligation to the Company at Stationer's Hall. "You madam, have a right to choose your friends," wrote Simpson, "but does it appear consistent with your obligations to Stationers Company to side with and join in ye cause of a avowed Enemy. . . . The *Diary* under his conduct was so scandalously abusive,[3] and grossly immoral, that ye Bishop of London (patron of ye Company) absolutely refused granting the License to print of it unless it was done by some other Hand . . . and had not ye Heads of ye Company waited upon his Lordship in a Body to procure his imprimation no *Diary* had come out that year. . . . "

One might at first be inclined to think that Thomas Simpson spoke from prejudice rather than justly, but *The Monthly Review* in speaking of Heath's publications has much the same criticism to offer of the character of his work. The articles in the *Ladies' Diary* during the years he was compiler constitute factual evidence against him, especially for the year 1752. In reviewing a work entitled *Astronomia Accurata: Or the Royal Astronomer and Navigator*, by Robert Heath, *The Monthly Review*[4] says,

"The author seems possessed of no very happy talent for explaining either his tables or calculations, so as to make them intelligible, by examples or precepts . . . and whoever has seen the *Palladium* (a short ephemeris by the same author) may thence form a just idea of *Astronomia Accurata* . . . so far as we have been able to understand him, by following his dictates, we must imagine he has set up his own opinion above the judgement of all who wrote before him, not even *Newton* and *Halley* excepted; . . . In many places he has so ill treated authors of great

[3] See De Morgan, *op. cit.*, p. 71.

[4] See *The Monthly Review*, February, 1761, v. 24, p. 153.

reputation, and inveighed against them with so much scurrility, and low abuse; that we chuse rather to refer the reader to the book itself, than defile our paper with transcripts. Here indeed his language is extremely perspicuous, and it seems to be the only strain in which he is properly qualified to write. This extraordinary author seems mightily afraid lest his book be pirated. . . . We imagine he might have spared himself the trouble, with regard to anything that is his own; and to some things he has manifestly helped himself to in other authors, without doing them the common justice to mention their names; those who have a mind to pirate will probably have recourse to these authors, than to Mr. Heath's book; seeing it is always best to deal with *principals*."

The trouble Simpson had with Heath, which caused him so much concern in connection with the *Ladies' Diary*, was the fact that Heath secured advance copies of material which was to appear in the *Diary*, and used some of it in his own publication. In spite of such disturbances Simpson made a success of his editorship of the *Diary*, and, as was his purpose, restored it to the character it had maintained under the guidance of Henry Beighton, and its mathematical parts were decidedly superior to those of Beighton's time. His editorship of the *Diary* brought to an end his life-long trouble with Robert Heath. In choosing questions and problems in mathematics, and in choosing the best solutions for publication, Simpson was educating a public which he would have been otherwise unable to reach. Often he made a particular effort to write individual contributors concerning their work, and if possible make helpful suggestions. Although the work served to bring Simpson's name before the public to an even greater extent than before, it in no way added to his eminence.

CHAPTER XII

Correspondence in Later Years

In the latter part of his life, Simpson entered into correspondence with several men who were outstanding mathematicians in England during the eighteenth century.

CORRESPONDENCE WITH JOHN LANDEN

John Landen was probably one of the most important of these correspondents. His chief claim to note through his publications is due to his *Mathematical Lucubrations* and his *Residual Analysis*,[1] which were published by subscription. Among the subscribers to the latter Simpson's name appears, together with the names of Mr. Waring, Fellow of Magdalene College, Cambridge; Mr. Richard Weston, Teacher of Mathematics in Stamford; Mr. Francis Maseres; The Rev. J. Colson, A.M., F.R.S., Lucasian Professor of Mathematics at Cambridge; Mr. Thomas Allen, Teacher of Mathematics in Spalding; and others whose interest in the publication was remote from the subject matter.

Landen was also responsible for several treatises in the *Transactions* of the Royal Society, and it was through his first attempt here that the friendship between the two men deepened. It was Landen's wish to present a paper to the Royal Society, a procedure which often resulted in the author being made a Fellow. Simpson had promised to present such a paper to the Society and to have it inserted in the *Transactions*. Much correspondence was necessary in this connection, and we find Landen writing in February 18, 1754, from Walton, near Peterborough enclosing the proposed paper:

"Sir,

"Inclosed is the Paper you was so obliging as to promise me you would present to your Society to be inserted in the *Philo-*

[1] See *Proposals for Printing*, Peterborough, 1758, to be found in the unpublished collection of Professor David Eugene Smith.

See *Discourse Concerning Residual Analysis*, London, 1758; *The Residual Analysis*, London, 1764.

sophical Transactions. I have put no Cover on it for fear of making my Packet too big: you will be so kind as to put one upon it if you think it necessary.

"With regard to the Use of my Theorems, I have to observe that, besides their peculiar Use in assigning the Fluents of such expressions as these:

$$\frac{w^m\,\dot{w}}{\frac{2n}{a+w} - 2y + \frac{n}{a+w} + \frac{2n}{a-w} + \frac{n}{a-w}} \cdot \frac{w^m\,\dot{w}}{\frac{n}{a+w} \pm \frac{n}{a-w}}$$

I am persuaded they will be found useful in other cases, wherein at first sight, they do not seem applicable. The Fluent of

$$\frac{\frac{n-2}{z+1} \times \dot{z}}{z^n + 1}$$

being required; if we substitute $\frac{1+w}{1-w}$ for z the Fluxion becomes

$$\frac{2^{n-1}\,\dot{w}}{\frac{n}{1+w} + \frac{n}{1-w}},$$

whose Fluent will I believe be more easily obtain'd by the help of Mr. Cotes's. Other instances of this kind may be given, but I shall add no more knowing that to a person of your discernment such would be quite unnecessary. I am with great esteem, Sir,

"Your most hble servt.

"J. Landen."

"P.S. A line from you to acquaint me with the receipt of this will be esteemed a favour. My compliments to Mr. Turner: I am sorry he missed my house in his last journey into the North."

One can surmise from Landen's next letter that Simpson was in some doubt as to the superiority of Landen's method over Cotes's. Writing the following April, Landen says,

"Sir; I must desire you to indulge me this Once in talking a little more of my last invention: I am pleased with it, and cannot forbear expressing the pleasure it gives me.

"You, in your last letter seem to doubt whether the conclusions deriv'd by the help of my theorems will be so simple and neat as those deriv'd by the help of Mr. Cotes's. I have, therefore, for your own private inspection, sent you a specimen of what may be done by the former; which specimen will, in some measure, enable you to form a comparison of the two Methods without your taking the trouble to compute Fluents by mine. The Theorems here sent, I find with much ease and perspecuity; and in the same manner, with very little more difficulty, (as will appear by my Lucubrations) may we reduce the Fluxions

$$\frac{\frac{p}{r \pm x} + \frac{q}{s \pm x} \&c. + \dot{x}}{\frac{n}{a+x} \pm \frac{n}{a-x}}, \qquad \frac{\frac{p}{r \pm x} + \frac{q}{s \pm x} \&c. + \dot{x}}{a^n \pm x^n},$$

$$\frac{\frac{p}{r \pm x} + \frac{q}{s \pm x} \&c. + \dot{x}}{a^{2n} - 2Ka^n x^n + x^{2n}}$$

and others, whose Fluents (I conceive) cannot be computed, by means of Mr. Cotes's Theorems, without excessive labour.

"I did not mean to communicate any Examples to the Society, unless you had thought it proper to do so, to give the Gentlemen some idea of the use of what is already communicated: And, if you think it necessary, you may, for that purpose, make use of what is here sent.

"Your civility to me I take extremely kind, and shall embrace every opportunity that offers to improve your acquaintance.

"I am, Sir, Your much obliged servt.

"J. Landen."

"Walton, April 15th, 1754."

"P.S. Mr. Cotes's Method succeeds in no operation better than in finding the Fluent $\frac{x^{m-1}\,\dot{x}}{a^n \pm x^n}$; yet even in that, many superfluous Terms are introduced, which destroy one another. In performing the same operation by my Method, by Circular Arcs being separated, in the easiest and most elegant manner, from the other part whose Fluent is assignable by Logarithms—My method

succeeds as well as many other enquiries: In short, I find the Invention of much greater use than I at first expected.

"When I come to town you shall see what artifice I make use of in applying the Theorems I sent you before to the investigating these I now send you."

The description Simpson writes of the occasion on which he read the paper, is well worth quoting in full:

"Sir,

"Your paper was read in the Royal Society on the 25 Day of ye last Month, and order'd to be printed in their next volume; of which I should have given you notice sooner, but I have been so ill of a cold that I got with being in Town, as not to be able to put pen to paper. I must inform you the celebrated Mr. Clairaut, of ye Royal Academy of Sciences happen'd to be then present who upon hearing ye preamble to ye paper read which was all that well could to such a mix'd hearing (ye most being of to difficult a Nature) immediately left his seat to enquire of me concerning ye Author, and ye Merit of ye performance; which I had ye more pleasure, as, this Gentleman has hinted in several Companys that ye English were, at this time greatly sunk in mathematical Learning. Since ye arrival of your last I have spent a few hours with a view to penetrate more clearly into ye merit & nature of your Invention (which I had not before considered, otherwise than by barely reading over your paper).

"The Method is certainly of much greater extent than might at first be imagined: The Fluents too are found and ye Alternate Terms separated with much facility. The reduction of ye Co-efficients seems to be ye most troublesome part of ye Work; but this, by restoring ye Co-sine (in ye room of ye Tangents) may be likewise perform'd I see without any great matter of difficulty and that by different Methods. In short ye whole seems to be a curious and fertile Invention; and such as must tend greatly to establish ye Author's Reputation.

"I am Sir, Your most Humble Servant, T. Simpson."

"P.S. The forms you point out would doubtless be a work of excessive Labour by Cotes's Method, without an Artifice to contract it."

This letter to Landen is by far the most interestingly written

one that has been preserved in the Simpsoniana. One written four year's later gives insight into Simpson's character to an even greater extent. After explaining the steps necessary to take to become a Fellow of the Society,[2] Simpson proceeds,

"You had no need to have made any apology for having communicated some Things that had escaped. I have not (if I may use ye words of ye celebrated author) ye vain ambition to want to shine alone or to imagine that I have exhausted all subjects on which I have turned my Thoughts. The method I have given in y^e^ *Philosophical Transactions* for ye Resolution of Hoperimetrical problems of that kind wherein only ye Fluxion of one of ye Two variable Quantities is concerned is not easily extended to cases where both fluxions have place. But there is a little Rule in my last book similar to what you lay down, whereby these last cases may be resolved; which may be applied with equal advantage however great ye number of Restrictions may be. This I discovered about a month after ye book was published, in consequence of a Hint mention'd by My worthy and ingenious Friend Mr. Rollinson, taken from ye end of McLaurin's Fluxions, to give you some idea of ye Reasoning I made use of to reduce all problems under that Rule. . . .

"I cannot put an end to this letter without mentioning to you one thing more tho' I am not certain that it will be quite proper. Mr. John Rowning (a Lincolnshire Clergyman) has wrote me (with a view I suppose to begin a correspondence) but has said some Things that give me Room to doubt that he is not a person of ye most exact Decorum (I should therefore be glad if when you write to me again you would give me some idea of his Character as I have at present no inclination to answer his letter,—[crossed out]). In one place he mentioned your name; where he says that when my Book of Fluxions first came out, he was pleased with ye 7th Section in particular; and that he thereupon wrote to a friend, in order to get Mr. Landen's Opinion of it, and to know whether it was new, or not, and that Mr. Landen returned for answer 'that it might perhaps be new, but that there was very little in it.' You will Sir, I hope excuse me for repeating to you so frivolous a Matter; which I cannot believe to be true; (Being

[2] See passage quoted above in Chapter VIII, page 137.

persuaded you would not speak worse of a thing than you thought it deserved, and your judgment is out of ye question,—[crossed out]). As I have always wished to be upon Terms of friendship with men of real Merit and Abilities, yourself in particular, anything having a contrary Tendency (however remote) would give some degree of concern, D^r^. Sir, to,

"Your obliged Humble Servant,
"T. Simpson."

"Woolwich Feb. 5, 1758."

How Landen received Simpson's letter concerning Mr. Rowning[3] is impossible to determine, but he was known to possess a violent temper, and might easily have resented it. Certainly he did not follow Simpson's suggestions as to methods of joining the Royal Society, and as he did not become a member until 1766, it is possible this situation terminated their friendship. Landen was a man of genius though restricted by his narrow education. His work was recognized both in England and on the continent during his life time. Simpson always was sensitive to criticism to the extent of vanity, which was perhaps due to his early history, for it is often true that those who fight their way up jealously guard their rights to an extreme. He was not, as a glance at his life of constant controversy might indicate, a naturally irascible man, but rather approached each new quarrel with feelings of sadness and of grievance. It is significant that Simpson never began a controversy but was always the defendant. Hutton's description of Simpson as a mild man could thus be reconciled with other existing evidence.

LETTER FROM JOHN ROBERTSON OF THE ROYAL NAVAL ACADEMY

A letter from John Robertson to Thomas Simpson recalls the fact that a Royal Naval Academy had been established at Portsmouth to meet the needs of the navy in a manner similar to the way in which the Woolwich Military Academy met the needs of the army. Of this academy John Robertson (1712–1776) became first master in 1755, having held a position as master of the Royal Mathematical School in Christ's Hospital since 1748. His life

[3] Author of two papers in the *Philosophical Transactions* on mathematical subjects.

in many ways resembles Simpson's. He was a man who became a mathematician although he had been first apprenticed to a trade. After his appointment to the Naval Academy he experienced much unpleasantness with the second master, and finally lost his appointment in 1766 "through petty cabals with the second master." He thereupon returned to London and was appointed clerk and librarian to the Royal Society on the seventh of January, 1768, which office he held until his death December eleventh, 1776. He was the author of many texts in popular use on navigation,[4] astronomy, and mathematics. He is said to have been the first to discover the theorem that in stereographic projection the angle between two circles on the sphere equals the angle between the two circles in the projection.[5] His friendship with Simpson was due to a mutual interest in mathematical subjects, as the following letter will show:

"Sir,

"I am sorry it so happened that your coming to Town and my leaving it fell so near to one another as to furnish but a part of one evening's conversation between us, especially as I had so many things that I wanted to discourse with you upon. I shall here mention one of them. The problem for finding the latitude for any time of day, the declination, two altitudes of the time between being given, has employed me some hours wanting to shorten the common trigonometrical operation which you know is to work in three spherical triangles: What induced me to think of it was some of our naval officers informed me there was a method used by the Dutch & from them, by some of our officers, that did not take more figures than used in two or 3 common

[4] *The Elements of Navigation*, 1754, which went through seven editions in fifty years.

A Compleat Treatise of Mensuration, 1739, of which there was a second edition in 1748.

Mathematical Instruments, 1747, of which the fourth edition was brought out in 1778 by W. Mountain.

A translation of De la Caille's *Elements of Astronomy*, 1750.

Nine papers in the *Philosophical Transaction* between 1750–1772.

[5] Chasles, *Aperçu historique sur l'origine et le developpement des méthodes en Géométrie . . . suivi d'un Mémoire de Géométrie, etc.*, Bruxelles, 1818, tom. 11, pp. 516–517.

Hutton, *A Mathematical and Philosophical Dictionary, etc.*, London, 2 vols., 1795, 1796. New edition, with many additions, etc., London, 1815, 2 vols.

proportions: I have not yet been able with any advantage to shorten the common method using those tables only in use among seamen which are the log of numbers, right sines, tangents, & secants. It any thing should occur to you whereby the problem may be solved, so as the Arithmetical work can be shortened, I shall be obliged to you if you will be so kind as to communicate it to me. A person ready in the use of the above tables, may work an example by the common method in about 30 minutes; but could it be shortened to 15 or 20 it would be much better. I am,

"Dear Sir
"Your most humble Servant,
"J. Robertson."

"Royal Academy, June 14, 1758."

LETTER FROM J. MICHELL, WOODWARDIAN PROFESSOR OF GEOLOGY, CAMBRIDGE

From Queen's College, Cambridge, Simpson received a letter containing a lengthy explanation of "reasons for supposing a retardation of the diurnal motion of the Earth arising from the Tides," which was sent by John Michell (1724–1793). This was the man who in 1762 became Woodwardian Professor of Geology at Cambridge. He was an astronomer of some note, but from the time of receiving the M.A. degree in 1752 he served the college officially as lecturer in Hebrew (1751–2, 1754, 1762), arithmetic (1751–2), geometry (1753–4, 1763). Doctor Bevis, a much mentioned friend of Simpson's, had told Michell of Simpson's interest and desire to hear of his researches. No more modest statement could have been made of them, even to the point of saying, "As you have already thought upon the subject I dont believe that I can say much, that shall be either new to you, or worth your hearing; . . . ", and offering as a simple apology for writing to Simpson when unknown to him, "as it gives me an opportunity of beginning an acquaintance with a man for whom, all lovers of mathematical and philosophical knowledge, cannot but have the greatest esteem." It seems difficult to believe that he was at one time President of the Royal Society and author of several very good papers which appeared in the *Transactions*.[6]

[6] See letter dated February 27, 1758.

CHAPTER XIII

Publication of "Miscellaneous Tracts"

Simpson's last legacy of any size to the public, was the publication of *Miscellaneous Tracts* in 4to, in 1757. The work was dedicated to the Earl of Macclesfield, the President of the Royal Society. "This was a most valuable bequest," says Hutton, "whether we consider the dignity and importance of the subjects, or his sublime and accurate manner of treating them."

1. The first of these papers is concerned in determining the *Precession of the Equinox, and the different Motions of the Earth's Axis, arising from the Attraction of the Sun and Moon.* The paper was drawn up about the year 1752 as a result of another on the same subject by Monsieur de Silvabelle,[1] a Frenchman. Although this gentleman had gone through one part of the subject with success and deliberation, and his conclusions conformed with Dr. Bradley's observations perfectly, nevertheless it appeared to Simpson that M. de Silvabelle had greatly failed in the most significant and only difficult part, that is in the determination of the momentary alteration of the position of the earth's axis, caused by the forces of the sun and moon, of which forces the amounts but not the effects were correctly investigated.

A letter written by Simpson November 15, 1752, indicates that he had submitted a paper giving some account of "M. de Silvabelle's Tract on the Precession of the Equinox"[2] to the Royal Society. This paper was later printed in the *Transactions* for the year 1754, a rough draft of which exists among his unpublished papers. In it Simpson says,

"The Author of this Tract begins, with a brief Enumeration of ye different Motions of precession and Nutation to which ye Axis of ye Earth is Subject, from ye Action of ye Sun and Moon on ye pertuberant Matter about ye terrestrial Equator. In which he delivers himself in so just and distinct a Manner that it is hard

[1] Guillaume de Saint-Jacques de Silvabelle (1722–1801), of Marseille, wrote on linear perspective, astronomy, etc.

[2] *Philosophical Transactions*, v. 48.

to conceive that anything better, or more proper could be said on ye same Occasion. It may therefore suffice here to give some Idea of his Calculations and ye Method he takes to determine the respective Quantities of those several Motions.

"In ye first place, in order to find ye Force whereby ye Earth tends to turn about its center, through ye attraction of ye Sun and Moon (on which ye whole depends), He conceives ye Earth to be circumscribed by a Spherical Surface, touching it at its Equator; and thereby forming a *double-Meniscus*, as it were; comprehended between ye said surface and *that* of ye Earth. He then proceeds to compute ye Sun's Force on ye matter capable of being included in ye Vacuity of ye said *Double-Meniscus*. This Force he considers as equal to that whereby ye whole Earth tends to turn about its Center: Because, was ye said *Meniscus* to be actually replete with Matter, of ye same common density with ye Earth, it is evident that ye Earth, being then a perfect Sphere, would have no Tendency at all to turn about its Center: through ye Attraction of any remote Body.

"Having in this Manner found the true Force whereby the pole of ye Earth tends towards ye plane of ye Ecliptick, from ye action of ye Sun, He proceeds next to determine y^e^ *Instantaneous* Alteration of ye position of ye Terrestrial Equator, in consequence of ye said Force.

"This is, by far, ye most difficult part of ye subject, which being once determined, every Thing that follows is purely Mathematical; nothing more being required than to take ye sum (Integral or Fluent) of these Instantaneous Alterations to have ye true Quantity of ye whole Alteration for any finite Time proposed. It may therefore be easily imagined that it is of much importance to have this point (on which everything depends) clearly settled; and that a Mistake here would be productive of very bad Consequences. Nevertheless it appears to me that our Author has passed this particular over without a proper degree of attention; so as to bring out ye result but about ye half of what it realy ought to be from his own principle.

"This I must confess staggered me very much for some time; especially as his Conclusions, however short of ye Truth they appeared were still found to exceed those of Sir Isaac Newton by near 1/3 part. I could easily see that the difference between

these Authors was chiefly (tho' not entirely) owing to ye different ratios assigned by them to ye greatest and least Diameters of ye Earth; and that this being allow'd for, ye Disagreement between them would remain inconsiderable. Notwithstanding which I cannot help thinking (as I have already hinted) that all Conclusions are too little by near one Half; according to ye principles and *Data*, from whence they are respectively deduced.

"I am very sensible that *this* may be deem'd too peremptory considering the authorities in question. But I must beg leave to observe that, had I not first consider'd the subject with all ye attention I was capable of, and satisfied myself in this Matter by Demonstrations which appear'd to me ye most clear and convincing, I should not have had ye presumption to say this much, with regard to an Author whose Merit is deservedly held in ye highest Veneration by ye very persons to whom I now have ye Honour to Address myself. Who would, I am convinced, be far from countenancing any unjust or little Attempt to ye prejudice of a Reputation which Reflects so much Honour on ye Society. As it was expected I should deliver my opinion without Reserve, on ye present Occasion, I could not avoid taking some Notice of an Authority so much in Favour of our Author: tho' repugnant to my own sentiments.

"After having considered ye Effect of ye Sun's Action, our Author goes on to determine that of ye Moon, according to ye different positions of her orbit with respect to ye place of ye Equator. Here He finds in a general Manner, the true proportion of ye Quantity of ye precession, depending on ye Moon to what it would be, if ye plane of ye lunar Orbit was to coincide with that of ye Ecliptick. He also finds ye true proportion of ye greatest Nutation of ye Earth's Axis to ye Mean Quantity of ye precession, in the Time of half a Revolution of ye lunar Nodes: and from thence infers that ye plane of ye Equator, during that Time, alters its Inclination to ye plane of ye Ecliptick by about 18 Seconds; exactly conformable to ye observations of *Dr. Bradley*. But this quantity nevertheless by reason of ye Mistake already hinted at, seems to me to be but little more than ye half of what it ought to be according to his own principles; And ye same thing may be said with regard to ye rest of his conclusions in general, so far as they relate to ye Motion of ye Earth's axis."

It was as a result of his conclusions with regard to M. de Silvabelle's paper and of the preparation of the above account for the Royal Society, that Simpson formulated the first tract of this publication.

2. The second paper contains the investigations of a very exact method or rule for finding the place of a planet in its orbit, from a correction of Bishop Ward's circular hypothesis, by means of certain equations applied to the Motion about the upper focus of the ellipse. By this method the result, even in the orbit of Mercury, may be found within a second of exactness, and that without repeating the operation.

3. The third paper shows the manner of transferring the motion of a comet from a parabolic orbit to an elliptic one; being of great use, when the observed places of a (new) comet are found to differ appreciably from those computed on the hypothesis of a parabolic orbit.

4. The fourth paper is an attempt to show mathematically that the mean of a number of astronomical observations is more exact than a single observation. This paper was first printed in the *Philosophical Transactions*, XLIX, (1755), Part I, pages 82–93, in a letter to "The Right Honourable George, Earl of Macclesfield, President of the Royal Society, on the Advantage of Taking the Mean of a number of Observations, in practical Astronomy." In this *Tract* Simpson has somewhat changed the original paper, but he gives the same two hypotheses that the distribution of errors might be a rectangle, each magnitude of error being equally probable, or that it might be an equilateral triangle, the probability of an error being inversely proportional to its magnitude.

5. The fifth paper contains the determination of certain fluents, and "the Resolution of some very useful equations in the higher orders of Fluxions, by means of the measures of angles and ratios, and the right and versed sines of circular arcs."

6. The sixth article treats of "the resolution of algebraical equations by the method of surd divisors; in which the grounds of that method as laid down by Sir Isaac Newton are investigated and explained."

7. The seventh paper contains a general solution of the isoperimetrical problem, with some examples of the use and application of the rule given. A part of the fourth paper together with

this one was inserted in the *Philosophical Transactions*, volume XLIX, but in this work several minor changes have been made.

8. The eighth paper and last, contains the solution of some general and important problems in mechanics and physical astronomy; in which, among other things, the principal parts of the third and ninth sections of the first book of Sir Isaac Newton's *Principia* are discussed. What may perhaps best recommend this tract is the application of the general equations thus derived to the determination of the lunar orbit. According to what Simpson had intimated at the end of the *Doctrine of Fluxions*, the greatest part of this "arduous task" was drawn up in the year 1750.

"The same objection, about that time made to Sir Isaac Newton's general law of gravitation, by Monsieur Clairaut of the Royal Academy of Sciences at Paris," says Simpson, "was a motive sufficient to induce me (among many others) to endeavour to discover, whether the motion of the moon's apogee on which that objection had its whole weight and foundation, could not be truly accounted for, without supposing a change in the received law of gravitation from the inverse ratio of the squares of the distances. The success was answerable to my hopes, and such as induced me to look farther into other parts of the theory of the moon's motion, than I first intended; but before I had completed my design, I received the honor of a visit from M. Clairaut (just then arrived in England) of whom I learned, that he had a little before printed a piece on that subject; a copy of which I afterwards received as a present at his hands; wherein I found most of the same things demonstrated, besides several others to which I had not extended my enquiry. . . . But I cannot omit to observe here, in justice to M. Clairaut that tho' he indeed fell into a mistake, by too hastily inferring a defect in the received law of attraction, From the insufficiency of the known methods for determining the effect of that attraction, in the motion of the moon's apogee, yet he was himself, the first who discovered the true source of that mistake, and who placed the matter in a proper light:"

Simpson mentioned the work of others who attempted to give the true quantity of that motion from such principles as are laid

down in the ninth section of the first book of the *Principia*, and of still others[3] who had explained it on other principles.

Simpson's own work was of an original nature and did not follow the methods of the continentalists. His method not only determines the motion of the apogee, in the same manner, and as easily as with the other equations, but utterly excludes the ambiguous terms that had embarrassed the greatest mathematicians, and that would, after a great number of revolutions, entirely change the figure of the moon's orbit. From whence this important concept is derived, that the moon's mean motion, and the greatest quantities of the several equations, will remain unchanged, unless disturbed by the intervention of some foreign or accidental cause.

Simpson used the analytic method as being the most direct and extensive and best adapted to this type of thought. Simpson also used a geometrical demonstration when he could, or where it seemed preferable, although he stated of himself, "I cannot be of the opinion of those who affect to shew a dislike to everything performed by means of symbols and an algebraic process; since, so far is the synthetic method from having the advantage in all cases, that there are innumerable enquirys into nature, as well as abstracted science, where it cannot be applied to any purpose."

Simpson closes his preface with a very significant statement:

"And it appears clear to me, that it is by a diligent *cultivation of the modern analysis, that foreign Mathematicians have, of late been able to push their researches further, in many particulars than Sir Isaac Newton and his Followers here, have done: tho' it must be allowed, on the other hand, that the same neatness, and accuracy of demonstration, is not every where to be found in those authors; owing in some measure, perhaps, to too great a disregard for the Geometry of the Ancients.*"

In this one sentence Simpson has summed up the situation which existed throughout the eighteenth century.

ELECTION TO THE ROYAL ACADEMY OF SWEDEN

Simpson's election to the Royal Academy of Sweden in 1758, recalls to mind the thirteenth paper in his volume of *Essays*

[3] See Walmsley's *Théorie du mouvement des apsides*, translated into English, *Philosophical Transactions*, Vol. 47, no. 11.

printed in 1742. His work on a general quadrature of hyperbolic curves, similar to that of Mr. Klingenstierna, Professor of Mathematics at Upsal, had so impressed the latter that when in later years he became secretary to the Academy at Stockholm, he was instrumental in having a diploma given to Simpson making him a member of that learned society.

At the time, however, when Simpson was admitted, Monsieur Wargentin was "Secrètaire pérpétual de l'Académie Royale dès Sciences de Suede." It was to him that Simpson addressed his acceptance of this high honor:

"Sir,

"The great Honour done me by the Royal Academy of Sweden in electing me a Member of their illustrious Society, demands my most grateful Thanks and Acknowledgment; which I most humbly beg of you Sir, to return them in my Name.

"The regard and approbation of a Body of Men eminently distinguished for their Improvements in every Branch of Knowledge, cannot fail of exciting my most ardent wishes to merit, in some degree, their good Opinion, by my future Endeavours, and to render myself, in the eye of the public, not altogether unworthy of the Honour conferred upon me.

"I am Sir,

"with most respectful regard,

"Your most obedient Humble Servant,

"Thomas Simpson."

"Woolwich,"

"Royal Academy, June 24, 1758."

"P.S. As I could wish to have it in my power to communicate any thing that might be worthy of the Notice of the Society I have been aiming at some little Improvements in the Method of Integration by infinite Series; which, at a proper Opportunity, I intend to take ye Liberty to transmit, for their Approbation."

CHAPTER XIV

Controversy with Muller

RELATIONS WITH MULLER AT THE ACADEMY

The disagreement between Simpson and John Muller culminated in 1758, and represented a situation of long standing. There was marked incompatibility between the two men from the start. Simpson was by far the better mathematician, but the regulations of the academy were such that for a great number of years he was made responsible to the first master, John Muller. Being a self-made man it was difficult for Simpson to take orders from any one, particularly from one for whom he had little respect. Muller, on the other hand, resented Simpson's superior accomplishments, and sought to exert his power at the Academy in order to make Simpson conscious of his higher office there.

Disagreement with regard to the geometry published by Simpson served to enhance the difficulty at the Academy. The situation centered about the question of the respective merits of their two textbooks.[1] The remarks which Simpson included in the second edition of his *Elements of Geometry* summarized his dealings with Muller up to 1760, but there are other interesting items which are even more enlightening to be found among his private notes. These show that Muller withheld "stationary ware," candles, and supplies, and in other petty ways sought to increase Simpson's irritability. It was not long before the students became aware of the lack of harmony, and in general the whole affair colored Simpson's career at the Academy with unpleasantness. Finally Muller requested that Simpson use his textbook in his classes, to which Simpson replied April 4, 1758, as follows:

"Sir,

"I received your Mandate, and in consequence thereof, beg Leave to represent to You, That, after having Myself wrote

[1] It was a question of the value of Muller's *Elements of Mathematics* and Simpson's *Elements of Geometry*.

Elements of Geometry, I cannot possibly submit to teach that subject by a Book of yours in which I am personally abused.—So degrading an Injunction, which I could not have expected, even from You, shews, as if you thought *this* a favorable opportunity to make me feel ye Effects of an inveterate, and unwarrantable Animosity; by which Y^e^ Business of ye Academy has long suffered.—But, be ye Event what it will (and it may not, perhaps, be exactly as you wish) you may be assured of this, that I shall never so far debase my own Character, as to conform to so opprobrious an Imposition.—There are, besides, some other Reasons, why I cannot think it, either proper, or for ye good of ye Academy, to teach Geometry by ye same Book, and in ye same Manner that you have done; but these it will be time enough to mention when call'd upon by proper Authority.—

"It is my design to resume that subject, as soon as I have ordered down Books and things proper to go on with.—But you will give me Leave, Sir, to think it odd that you should, now, talk about ye Trouble of Beginning Geometry every Winter! Was it not your own choice? You had Books of Geometry to sell.—Had you not interfered in a Subject that did not belong to you, I should never have discontinued it.

"Tho. Simpson."

"Apr. 4, 1758."

Evidently even the course in geometry at the Academy was discontinued for a time as a result of the incompatibility of Simpson and Muller, and their quarrel regarding the respective texts. The quarrel was finally taken to the Ordnance office with the result that Simpson wrote letters in appeal to their request that he agree with Muller:

"My Lord,

"Col. Desaguliers signified to me, 'that it is your Lordship's pleasure that I should agree with Mr. Muller.' As I do not my Lord think that this amounts to a positive order, for me to submit without ye Liberty of an Appeal, to whatever Mr. Muller may please to impose upon me, I most humbly beg leave to lay before your Lordship some reasons why I conceive ye good Ends aimed at by your Lordship will not be attainable by ye measure which ye first Master seems determined, at all events, to support by his Credit with your Lordship.

"I shall not here insist on ye mean and scandalous abuse with which I am unjustly loaded in that very Book he now enjoins me to teach by, tho' that alone is sufficient to render a good Agreement between us utterly impossible: I have other reasons to offer which may weigh more with your Lordship, and which Mr. Muller himself does not seem to have well considered.—It will be impossible for me to teach Geometry by his book with one half ye Dispatch and perspecuity that I can otherwise do by being left at Liberty to make use of my own judgment, and to act in ye best manner I am capable of. But this is not all, I cannot use his Book without being under an absolute necessity of exposing ye Author's want of judgment, from ye multitude of Inaccuracies, Blunders, Errors, and Absurdities with which ye Book abounds, (and which must occur and be taken notice of in ye teaching of it [crossed out]). This, my Lord, would not only have a bad Effect in ye Academy, by lessening his Credit there, and discouraging ye Young Gentlemen in ye study of his other Works, which they are oblig'd to read, but would also furnish a new Source of Contention and endless Matter of Complaint.

"I could my Lord urge other Reasons why I humbly apprehend that it will not be for ye good of ye Institution that I should be compelled to teach Geometry by Mr. Muller's Book (notwithstanding his specious pretences to sell his book in ye Academy and to set up a new kind of Tyranny over ye other Masters), but I dare not intrude on your Lordship's patience. I am,

"My Lord,
"Your Lordship's most obedient
"and most Humble Servant,
"T. Simpson."

"Royal Academy,"
"Apr. 17, 1758."

The following letter may have been intended as a more complete appeal on the same occasion:

"My Lord

"As it would give me inexpressible concern to appear before your Lordship in ye Light of a Criminal, and having great Reason to be apprehensive, that Mr. Muller will not fail to use his utmost credit with your Lordship, to represent an Affair that has lately

happened between us in ye most disagreeable colours, and to draw upon me your Lordship's Displeasure; I do, therefore most humbly beg leave to be heard by your Lordship, in my own defence.

"About 10 Days ago Mr. Muller signified to me by a Billet, that '*it was his order that I should teach Geometry by ye same Book and in ye same manner that he had done.*'—As I had never before my Lord received an order, or Directions from Mr. Muller or any man living how to teach, and as ye Books wrote by myself for ye use of ye Academy have been extremely well received by ye publick and have been translated into different languages, I could not help looking upon that order to teach by a Book of his writing wherein I am personally abused, as a very great insult, and as ye utmost effort of a designing, and inveterate enemy, who, relying on his credit with your Lordship (to whom I have the misfortune to be unknown, [crossed out]) seems to look upon this as a favorable Opportunity, either to force me to submit to ye most ignominious Impositions, or else to procure my Dismission.

"It is not to be doubted that he will endeavour to persuade your Lordship into a Belief, that it will be for ye Benefit of ye Academy that his Book shou'd be taught there. But, whatever his pretensions are, his pride and his own Interest (by ye sale of ye Book) are at ye Bottom. And I shall demonstrate to your Lordship that this Book, which I am so presumptorily commanded to make use of, is really a very bad one and not at all fit for ye Academy.—It would be impossible, my Lord, in ye compass of a Letter to lay open, even one-tenth part of ye Inaccuracies, mistakes, and Falsehood contained in this Book, and (if it could be done) might trespass too much upon your Lordship's time and patience; for which reasons, I must here confine myself to ye pointing out of a few of ye most palpable ones.

"In his 7th Theorem (p. 12) he says that, 'If two Triangles have one side equal each to each, and any two angles in ye one are equal to any two angles in ye other, each to each, they are equal in all respects.'

"Now, that this Theorem is false in many cases, will appear from ye Triangles *ABC DEF* here annexed; wherein ye angles *D* and *E* being supposed, respectively equal to ye angles *A* and *B*, and ye side *DF* equal to ye side *CB*, the Triangles themselves will

nevertheless be unequal (contrary to what he asserts) except when ye angles *A* & *B* are also equal to each other.

"Again, at page 58 He asserts, 'That two circles will cut each other when ye Distance between their centers is either less than y^e^ Sum, or greater than y^e^ difference of their Radii.

"But this is also false, as y^e^ schemes here exhibited clearly evince. In Fig. 1 y^e^ lesser circle *BFGH* does not cut y^e^ greater

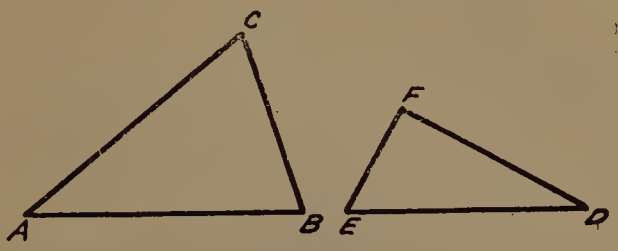


FIG. 19

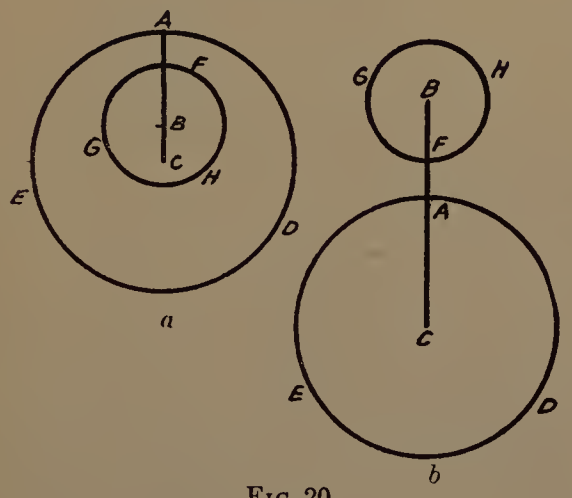


FIG. 20

CEAD, but falls wholly within it, notwithstanding y^e^ distance *BC* of y^e^ centers is much less than y^e^ sum of ye radii *CA* and *BF;* and, in Fig. 2, ye lesser falls wholly without y^e^ greater, tho' y^e^ distance of their centers is much greater than y^e^ difference of y^e^ Radii *CA* and *BF;* all which are utterly repugnant to his pretended Demonstrations.

"Again, in his 10th Theorem (at p. 35) He affirms that, 'The Sums of proportionals are proportional, or that if $A:B::C:D$,

and $E:F::G:H$ then will $A + E : B + F :: C + G : D + H$. But this, even by common numbers may be proved to be false. Let, for Instance y^e^ two ranks of proportionals be 1, 4, 3, 12, and 4, 6, 8, 12; then y^e^ Sums of y^e^ corresponding terms will be 1 + 4, 4 + 6, 8 + 3, and 12 + 12 (or 5, 10, 11, & 24): which are in no sort proportional, as this author asserts. His theorem is indeed true in that single case, where all ye ratios are equal. But by his using it in his conic sections (at p. 147) where ye ratios are unequal, it is clear that he look'd upon it to be universally true. He may perhaps pretend that these are oversights only, were they allowed to be such (which cannot be done) would be quite unpardonable in ye most rigid and accurate of all sciences; and are expressive in a high degree, of a confused Imagination; ye Distinguishing Characteristick of this Author.

"But bad as this Book is, his manner of teaching it (which I am also commanded to follow) is still worse. This will sufficiently appear from ye progress ye Young Gentlemen have made under his Tuition, during ye last 6 Months; not one of whom has acquired so much Geometry as might be learned in six days with proper Instructions: The greater part of them have no Idea at all of ye Subject, not even so much as to distinguish an angle from a Space: and yet *These* are permitted to say Geometry by rote, out of Mr. Muller's book, even in ye presence of your Lordship. This will probably startle your Lordship (who have ye good of ye Academy so much at heart). But your doubt my Lord may easily be removed by ordering a General Examination, by a proper disinterested person. Nay, I am almost persuaded that were your Lordship only to give your Word, that nobody would Suffer for Speaking ye Truth, the Young-Gentlemen themselves would not hesitate a moment to confess to your Lordship that they know nothing at all of ye Matter. (They even go so far as to make a jest among themselves of their being able to say their lessons (as they term it) without having any notion at all of ye Subject,—[crossed out])

You may my Lord very easily satisfye yourself, as to ye truth of what I have advanced. I know too well what is due to a person of your Lordship's Rank and Discernment to attempt to deceive by false Representations, and have more regard to my own Character, than to take such a Step as must inevitably draw

upon me your Lordship's Resentment, at a time when it is my utmost wish and ambition to merit your Lordship's Countenance and Protection,
I am,

"My Lord, with ye profoundest
"Deference & Respect, your
"Lordship's most obedient servant."

"P.S. It would ill become me to dictate to your Lordship; but, if it were order'd, that Mr. Muller and I should each of us propose a few problems in ye different Branches of Science to ye other to see which could give ye Best Solutions to them, I believe that your Lordship would not discover an over-willingness in ye man who insults me under ye Authority of his Post, to put ye Event upon that head.—He has hitherto abused me, with impunity, because I thought him below my notice, as a Mathematician, and because my Friends have persuaded me that my character was out of his reach to hurt. But this last effort of an unprovok'd and inveterate Enemy, Aims at something farther than ye Traducing of my Character; my repose and my future well being are now deeply struck at, by his Endeavour to make your Lordship's power useful to his Designs. I can, therefore, no longer avoid making use of ye proper means to disarm so determined an Adversary. And I doubt not to make it appear to your Lordship and to all ye World, not only that ye Book he commands me to teach by is really a desirable performance, But that his other publications which are crude, indigested Collections, abounding with innumerable errors and absurdities; and that his Construction of Bridges, his theory of Gunpowder, and some other Things, on which he most magnifies himself, are not only built upon false principles in Mechanics, but that ye Geometrical Reasoning and Demonstrations are also false.

"Ostentation, my Lord, was never reckoned in ye Number of my Faults; but an Over-Modesty, at this Time, would be a Fault.—I have ye Honour to be known to My Lord Macclesfield, My Lord Charles Cavendish, My Lord Willoughby of Parham, and to many other persons of ye greatest consideration in ye Royal Society; any of whom I am persuaded, would give your Lordship some Idea of my Character."

Judging from Simpson's remarks in the preface to his second

edition of the *Elements of Plane Geometry*, some such examination by the Master-General of the Ordnance was conducted, with the result that the clause in the regulations of the Academy requiring the "Second Master" to "teach by the method of the First Master" was changed. This would not seem to have any lasting effect, for a letter dated January 29, 1760, written by Simpson, would indicate that this matter regarding texts and methods of teaching geometry was a persistent source of contention between Muller and himself.

"Honourable Sir:

"The matter which I here presume to lay before you, nearly concerns ye Good of an Institution for whose prosperity and reputation You are solicitous; and claims ye attention of Those who have the power to rectify and redress.

"By a positive order of ye late Master General, the Teaching of Geometry in ye Royal Academy of Artillery, is expressly allotted and enjoin'd to the Second Master: Notwithstanding which, the First Master, for Reasons best known to himself, and by an intire neglect of his own proper Duty, has lately taken upon himself to teach that Branch; and that by a Method which in consequence of an actual Examination had been judged unfit and set aside by the Authority of the Master General.

"The ill effects of such a proceedure, and the confusion that must be produced in Young minds by being embarrassed between the different methods need not be pointed out to you sir: permit me only to declare to you that, from ye hour he laid down his own branch of Fortification, to take up ye Teaching of Geometry, a general discontent has prevailed in ye Academy.—But tho' I think it my Duty to promote to ye utmost of my power, the good of an Institution under which I am placed, and tho' my endeavours have been in some measures frustrated by the irregular conduct and Tyranny of the First Master; I should not sir, have appealed to you where it not for his despotic management, one instance of which is seen in his withholding supplies of stationary allow'd for ye other masters. So far from being able to obtain from him a Quantity sufficient for ye Academy, I have frequently been obliged to buy for my own use The Almanacks, penknives annually allow'd to ye masters in common with ye clerks and other servants of the Board. I would not have presumed to trouble you on this

Head, if my own Character had not been at stake, so as to give me an undoubted right to apply for redress to ye Justice of my superiors. For, I most humbly beg leave to appeal to you, Sir, whether his laying aside his own Branch to take up that of ye second Master, does not strongly suggest to everyone a want of Ability in ye Second Master to discharge his Duty in a proper manner? and whether the propagating of such an opinion, will not become a necessary step for ye First Master to take, in order to give a colour to his own unwarrantable proceedings.

"From these different considerations, and from ye known attention of the Master-General and ye Honourable Board to whatever concerns ye publick good, I have great reason to hope that a stop will be put to ye disorder that now subsists in ye academy, and that such directions will be given, as shall enable ye Masters to exert themselves with success in a cheerful and uninterrupted discharge of their Duty.

"I am,
"Honourable Sir,
"Your most humble
"and most obedient Servant,
"T. Simpson,
"Second Master."

"Woolwich,"
"Royal Academy,"
"January 29, 1760."

The trouble between Muller and Simpson persisted up to the time of Simpson's death. Perhaps the first decision of the Master General of the Ordnance would have proved satisfactory but with a change in administration, Muller began another quarrel which was in substance a continuation of the disagreement over geometry, and made it necessary for Simpson to appeal again to the Ordnance office.

CHAPTER XV

Final Years

ILLNESS

Thomas Simpson was subject to periodic illnesses throughout his life, but he was more and more indisposed in later years. Although in appearance he was tall, large, and apparently strong bodied, at fifty years of age he had become extremely aged. He had never been strong constitutionally; he was always very nervous, and the life he led by applying himself too closely to his studies both day and night and taking an insufficient amount of exercise gradually made his condition more critical. Hutton mentions the fact that he received improper nourishment through the neglect and extravagance of his family. Stories of his wife's ill temper and barbarous treatment of him are perhaps greatly exaggerated, for in contrast Hutton represents Simpson as a man "so gentle, mild and placid," which hardly seems possible in the light of his evident ability to dispute. Hutton says of Mrs. Simpson, "While she was revelling in the parlor with her family and gossips, he was banished to the garret to toil at his book, with no other supper than thin milk from which she had skimmed the cream for her tea and accompanied with an order that he should not be seen below that night. . . . "

To stories of Simpson's frequenting poor company with whom he drank porter and gin, Hutton remarks that it must be remembered that "the misconduct of his family put it out of his power to keep the company of gentlemen, as well as to procure a better liquor." Perhaps also Simpson never really outgrew the class into which he was born.

WORK ON THE NEW BRIDGE AT BLACK FRIARS

It was during the year 1760 when his illness was most critical, that Simpson applied himself diligently to the selection of the best form of arches for the new bridge to be constructed at Black Friars. During this agitation, Simpson among others, was con-

sulted upon the best form for the arches by the "New-bridge Committee." He gave preference to the semicircular form; and besides his report to the committee, some letters also appeared on the same subject, by himself and others, in the public newspapers and particularly in the *Daily Advertiser* and in *Lloyd's Evening Post*. These letters were collected into the *Gentleman's Magazine* for that year.[1]

FURTHER WORK ON BRIDGE CONSTRUCTION

It is probable that this reference to bridge construction turned Simpson's thoughts more seriously to the subject, for he considered writing an elaborate treatise on it. "His family," says Hutton, "have often informed me that he laboured hard upon this work for some time before his death, and was very anxious to have completed it, frequently remarking to them that this work, when published, would procure him more credit than any of his former publications. But he lived not to put the finishing hand to it."

REMOVAL TO MARKETBOSWORTH AND DEATH

Exercise and proper regimen were prescribed to restore Simpson to good health, but finally he was unable to perform his duties or even read the letters of his friends. His trouble with John Muller may have greatly increased his disturbed condition, for it was just at this time that Muller renewed the quarrel.

Finally physicians advised his native air for his recovery. In February, 1761, he set out with much reluctance, believing he would never be able to return. The journey to Bosworth in the company of a few of his relations so fatigued him that upon his arrival there he took to his bed, where he grew continually worse, until the day of his death May 14, 1761.

SURVIVING MEMBERS OF HIS FAMILY

He was survived by his wife, her son and daughter by her first husband, and his own daughter and son, the latter of whom was born after his removal to London. The two daughters had married officers in the artillery. Young Simpson had attended

[1] See *Gentleman's Magazine*, London, 1760, pp. 143, 144.

school to his father, and upon special application by the latter, had received a commission in the corps. He was given a company in the Royal Artillery, but lost his post because of drunkenness and other misconduct.

Simpson was devoted to this son and was especially proud of his admittance to the cadet-company. In a letter written in his behalf Simpson describes young Thomas in glowing terms: "I have been emboldened to take this liberty in behalf of my only son, a Youth of fifteen; who is very desirous of being Admitted into the Cadet Company: He has attended the Academy a considerable Time, and is inferior to none of his years, in the several branches there taught . . . if at any time a vacancy happens, it would ill become me to be importunate for the Grant of a Favour that must intirely depend on your honour's goodness, but I should be so happy as to be Thought of by You. . . . "

Scarcely a year later he again writes concerning him, having received a favorable reply to the previous communication:

"As your Honour, on the Application of Captain Farrington in behalf of my son, was so good as to say, he should be provided for, I am encouraged to Move ye Matter again, at this Time, to your Honour.

"The Youth for whom I intreat your Honour's Regard, is near Sixteen; has attended ye Academy for several Years; and besides ye knowledge there acquired, is, in some Degree, acquainted with almost every thing done or practiced in ye Regiment; he having a strong Bent to Military Concerns, as well from Nature as Habit. . . . "

Still later Simpson refers to him in a letter to a friend, saying, "Tom is with ye Regiment of ye old Buffs, at Somerton, in Somersetshire, if he be not gone from thence to encamp which (by his last letter) he intended to do, in a very short Time."

As for the widow, the King, at the request of Lord Viscount Ligonier, the then Master General, in consideration of Mr. Simpson's merits, was graciously pleased to grant her a pension for life, together with apartments adjoining the Academy; a favor that never had been conferred upon anyone before. Here she lived with her son Swinfield and his wife, in the same apartments, until her death December 14, 1782, at the age of 102. Her son Swinfield was still alive July 28, 1792, and was supported by

a weekly allowance from some relations. It was from Simpson's wife and from her son Swinfield that Charles Hutton was able to get his account of Thomas Simpson's early life.

DISPOSITION OF SIMPSON'S PAPERS

Whatever Simpson wrote upon the subject of bridges, together with all his other remaining papers, fell into the hands of Major Henry Watson, of the Engineers, in the service of the India Company. His remaining papers were of great number, being enough to fill a large chest, according to Hutton's account. Henry Watson had been a pupil of Simpson's and had lodged at his house. After Simpson's death, Watson prevailed upon Mrs. Simpson to let him have the papers, promising either to give her a sum of money for them, or else to print and publish them for her benefit. Nothing of the kind was ever done, and Watson upon being questioned closely by Hutton, always declared that no use could be made of any of the papers, owing to the very imperfect state in which Simpson had left them. Yet he continued to refuse to give them up.

The private papers that I have made use of in this study are without doubt those of which Henry Watson gained possession, but if there was any material regarding his work on bridge construction, it has been lost. They were acquired for the celebrated Phillips collection and were then sold to Professor R. C. Archibald of Brown University (Providence, R. I.) by whom they were given to Professor David Eugene Smith.

CHAPTER XVI

Summary

POPULAR INTEREST IN MATHEMATICS AFTER 1730

The period in which Thomas Simpson was an outstanding mathematician and educator, was one of semi-isolation of Great Britain from the continent in mathematics. While historians in general have dismissed the time as lacking both in interest and significant development, the account of Thomas Simpson's career reveals that there were many important but unorganized educational forces at work, of which one of the most important was the development of a popular interest in mathematics among the British. In an age when education was regarded as the concern of the individual, self-teaching and private-tutoring, as well as friendly aid given in mathematical clubs were important means of instruction. Thus it was not so much the schools or educational institutions which determined the type of mathematics studied, as it was the scientific correspondence, the mathematical periodicals, and the elementary treatises and textbooks. The new reading public demanded a simplified presentation of subject matter, which increased the literary productivity of the period.

Throughout his life Thomas Simpson contributed more than any other one man toward the development of this popular interest. As a member of the two characteristic types of scientific societies, the Royal Society at London and the mathematical club at Spitalfields, he came in touch with all classes of men whose interests were mathematical. In correspondence he reached many people who were either financially or geographically so handicapped as to be unable otherwise to secure instruction. As contributor, and later as editor of the *Ladies' Diary*, the most popular of the mathematical periodicals of the eighteenth century, Simpson was able to raise the standard of the journal and to exercise his abilities in obtaining a wise selection of material for the readers. In this capacity he was able to give voluntary instruction to contributors as the need arose, and thus to reach an

even greater number through his teaching. His publications were generally written with the novice in view, and his treatises and textbooks were important because of the attempt to simplify the presentation of subject matter. The wide range of material included in his publications is representative of the interests of the time.

THE DOMINANCE OF NEWTON OVER MATHEMATICAL THEORY AND INTEREST

The dominance of Newton over the mathematical theory and interest of the later Newtonian school to which Thomas Simpson belonged is at once apparent in this study of Simpson's life, writings, and correspondence. Newton's influence was expressed in the work of later writers in their choice of subject matter and in the use of the geometric method of proof. Simpson's correspondence with Francis Blake is illustrative of the influence of Newton's *Principia*, which gave rise to the popularity of such topics as the theory of attraction, the application of the principles of mechanics to the solar system, the creation of physical astronomy, and the establishment of the law of gravitation, as well as particular questions connected with the earth and moon, and the theory of hydrodynamics.

At various times throughout his life Simpson produced essays which sought to make Newton's meaning clear to the elementary student of mathematics. In addition he carried Newton's researches even further as in the case of his *Miscellaneous Tracts* (1757) where he discusses the material in the *Principia* concerning the lunar orbit, and gives the differential equation for the motion of the apse of the lunar orbit similar to that arrived at by Clairaut, a French contemporary. His results, obtained by deducting a general solution by indeterminate coefficients instead of through the use of successive approximations, enhances the value of this work. In another tract in this collection, Simpson has attempted to show mathematically that the mean of a number of astronomical calculations is more exact than a single observation, and he has given what represents one of the earliest attempts in statistics, that is, a discussion of the formulas of facility of error. Simpson also showed discernment in making use of the algebraical method

of analysis in his publication of *Miscellaneous Tracts*, and in attributing progress on the continent to the use of this method.

Newton's influence is also apparent in a study of Simpson's contributions to the doctrine of fluxions. Simpson, as well as many of his British contemporaries, followed the fluxional calculus and the cumbersome notation of Newton, rather than that of Leibniz which was developed on the continent. Simpson on the whole avoids the confusion expressed by many writers on the subject, due to the superposition of British symbols and phraseology upon the older continental conceptions. He avoids the use of infinitely small quantities, as did Newton in his *Quadratura Curvarum* (1704). Their use by British writers was one of the objections to the calculus voiced by Berkeley in the *Analyst* (1734). Simpson treated the doctrine of fluxions as closely associated with geometry to the neglect of analysis, a further indication of the dominance of Newtonian conceptions over later eighteenth century thought. From the standpoint of rigor his method was far in advance of the continental.

In addition, the combined efforts of Thomas Simpson and his friend Francis Blake produced an essay which was a real contribution to the logic of fluxions and to an understanding of Newton's work.

SIMPSON'S CONTRIBUTIONS TO INSURANCE THROUGH THE DOCTRINES OF CHANCE, AND OF ANNUITIES AND REVERSIONS

Simpson's work on the *Nature and Laws of Chance* (1740) and his later publication on the *Doctrine of Annuities and Reversions* (1743) represent pioneer efforts in fields which previously had been little developed with regard to the inexperienced reader. A custom of the eighteenth century was to publish results without method. Simpson, however, wrote for the lay mind, and in addition made clear the work of others in these fields of mathematics. The tables for the valuation of annuities for single and joint lives taken from the London bills of mortality represent an early contribution in the development of insurance. These tables were in use by the two great insurance companies, the Equitable and the Amicable Insurance Societies, during the eighteenth century.

SIMPSON'S EFFORTS IN TEXTBOOKS ON ALGEBRA, GEOMETRY, AND TRIGONOMETRY

Simpson's contributions in the elementary textbook field extend to the subjects of trigonometry, geometry, and algebra. He had a peculiar capacity for simplifying and clarifying subjects which had been presented in an abstruse and difficult way. In most respects his textbooks in these subjects are representative of the best the century had to offer, although deficient in the light of the present day. His textbook on algebra, while explanatory to a fault, is more brief than many of the works of his contemporaries. He has included, however, a great amount of irrelevant material which has survived in our textbooks up to the present day. Simpson was one of the few of the eighteenth century textbook writers to advocate a change in the order and arrangement of material as given in *Euclid*. His work on geometry is a pioneer effort in the English language in the development of an elementary textbook on geometry to replace *Euclid* without the sacrifice of rigor. His *Trigonometry*, noteworthy for its brevity, contains the current abbreviations for the trigonometrical functions and constitutes an improvement over contemporary textbooks. The tables of logarithms he has included in this work were in popular use and were copied by later writers.

SIMPSON'S CAREER AT WOOLWICH MILITARY ACADEMY

During his career as professor of mathematics at Woolwich Military Academy (1743–1761), Simpson became one of the scholastic fathers of all the great engineers that England produced over a period of almost twenty years. As one of the first professors of the corps of gentlemen cadets he devoted his energies to perfecting the course of study offered at the Academy, as well as to creating a high standard for the entrance requirements to that institution. The high standard of the mathematical training offered at Woolwich during the eighteenth century is in part due to his labors.

SELECTED BIBLIOGRAPHY

Allibone, Samuel Austin. *A Critical Dictionary of English Literature and British and American Authors Living and Deceased from the Earliest Account to the Latter Half of the Nineteenth Century.* J. B. Lippincott Company, Philadelphia, 1892.

Backhouse, W. *A Dissertation on the Value of Life Annuities . . . Applied to the Schemes of the Laudable and Amicable Societies . . . With tables, etc.* London, 1778.

Ball, W. W. Rouse. *A Short Account of the History of Mathematics.* 6th ed., London, 1915.

——— *A History of the Study of Mathematics at Cambridge.* Cambridge, 1889.

Barrow, Isaac. *Euclidis Elementorum Libri XV.* Cambridge, 1655, with many later editions.

——— *Euclidis Data.* Cambridge, 1657.

Betham, William. *The Baronetage of England, or The History of the English Baronets and Such Baronets of Scotland, as Are of English Families; with Genealogical Tables; and Engravings of Their Coats of Arms. . . .* Printed by Burrell and Bransby, for Longman and Rees, and Robinsons, 1801–05.

Berkeley, George. *The Analyst.* London, 1734.

Blake, Francis. *An Explanation of Fluxions in a Short Essay on the Theory.* London: Printed for W. Innys, at the West-End of St. Paul's, MDCCXLI. Published anonymously.

Bombelli, Rafael. *L'Algebra parte maggiore dell' aritmetica divisa in tre libri.* Bologna, 1572.

Bonnycastle, John. *An Introduction to Algebra, with Notes and Observations . . . to Which Is Added an Appendix on the Applications of Algebra to Geometry. . . .* 1st, New York, from the 10th London ed. . . . New York, E. Duyckinck, 1818.

——— *An Introduction to Algebra; with Notes and Observations Designed for the Use of Schools and Places of Public Education.* The 1st American edition. . . . J. Crukshank, Philadelphia, 1806.

Bradwardine, Thomas. *Arithmetica Speculatiua.* Paris, 1495.

——— *Tractatus de proportionibus.*

——— *Geometria speculativa.*

——— *De quadratura circuli.*

Brand, Charles. *A Treatise on Assurances and Annuities on Lives. With Several Objections Against Dr. Price's Observations on the Amicable Society and Others, etc.* London, 1775.

——— *A Table of Lives in the Corporation of the Amicable Society for a Perpetual Assurance-Office from the Institution of the Society in 1706 to the Year 1777.* London, 1778.

British Museum Catalogue.

CADELL, T. *Elements of Algebra.* London, 1779.

CAJORI, FLORIAN. *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse.* Open Court Publishing Company. Chicago and London, 1919.

——— *The Teaching and History of Mathematics in the United States.* Washington, 1890.

CARDAN, H. (or J.). *Artis Magnae, sive de regvlis algebraicis, liber vnvs.* Nürnberg, 1545.

CANTOR, M. *Vorlesungen über Geschichte der Mathematik.* 4 vols., Leipzig, 1880–1908, with various revisions.

CHÂSLES, MICHEL. *Aperçu historique sur l'origine et le developpement des méthodes en Géométrie . . . suivi d'un Mémoire de Géométrie, etc.* Tom. 11, Bruxelles, 1818.

CHEYNE, GEORGE. *Fluxionum methodus inversa: sive Quantitatum Fluentium leges generaliores.* (Addenda et adnotanda, etc.) Londini, 1703. The "Addenda" are separately paged.

CLAIRAUT, ALEXIS CLAUDE. *Elémens d'Algèbre.* Paris, 1749.

——— *Elémens de Géométrie.* Paris, 1765.

CLARK, SAMUEL. *The Laws of Chance: or a Mathematical Investigation of the Probabilities Arising from Any Proposed Circumstances of Play, etc.* London, 1758.

——— *A Letter to Richard Price . . . Containing an Entire Refutation of His . . . Treatise of Observations on Reversionary Payments, etc.* London, 1777.

Daily Gazetteer, The. London, 1737.

DEMOIVRE, ABRAHAM. *The Doctrine of Chances: or, A Method of Calculating the Probability of Events in Play.* London, 1718.

——— *Annuities upon Lives; or, the Valuation of Annuities upon Any Number of Lives; as, Also, of Reversions.* To which is added, an appendix concerning the expectations of life and probabilities of survivorship. London, 1725.

DEMORGAN, AUGUSTUS. *A Budget of Paradoxes . . . Reprinted, with the author's additions, from the "Athenaeum."* Edited by S. E. DeMorgan. London, 1872. Second edition, edited by David Eugene Smith, The Open Court Publishing Company, Chicago, 1915.

——— *Arithmetical Books; from the Invention of Printing to the Present Time: Being Brief Notices of a Large Number of Works, Drawn Up from Actual Inspection.* London, 1847.

DESCARTES, RENÉ. *La Géométrie.* C. Angot: Paris, 1664.

——— *Le Monde de Mr. Descartes ou le traité de la lumière, et des autres principaux objets des sens. Avec un discours du mouvement local et un autre des fièvres, composez selon les principes du même auteur.* Paris, 1664.

DITTON, HUMPHREY. *An Institution of Fluxions: Containing the First Principles, The Operations, with Some of the Uses and Applications*

of That Admirable Method; According to the Scheme Prefix'd to His Tract of Quadratures, by (its first Inventor) the Incomparable Sir Isaac Newton. By Humphrey Ditton, London, 1706.

Dodson, James. *The Mathematical Repository, etc.* 3 vols., London, 1748–55.

Donn, Benjamin. *The Accountant and Geometrician: Containing the Doctrine of Circulating Decimals, Logarithms, Book-Keeping and Plane Geometry, etc.* 4 pt., London, 1765.

——— *Mathematical Essays; Being Essays on Vulgar and Decimal Arithmetic . . . With a General Preface, Including a Panegyric on the Usefulness of Mathematical Learning.* London, 1758.

Emerson, William. *The Doctrine of Fluxions, Not Only Explaining the Elements Thereof But Also Its Application and Use in the Several Parts of Mathematics and Natural Philosophy.* Third edition, corrected and . . . enlarged. London, 1768. First Edition, London, 1743.

Encyclopaedia Britannica, XII ed.

Gentleman's Magazine, The; or, Monthly Intelligencer. Vols. 1–5, January, 1731–December, 1735. (Edited by E. Cave from 1731 to 1754.)

Gentleman's Magazine, The; and Historical Chronicle. Vol. 6–77, January, 1736–December, 1807.

Graves, Robert Perceval. *Life of Sir W. R. Hamilton . . . Including Selections from His Poems, Correspondence and Miscellaneous Writings,* 1882.

Hammond, Nathaniel. *The Elements of Algebra . . . to Which Is Prefixed an Introduction, Containing a . . . History of This Science.* London, 1742; 3rd ed., 1764.

Holliday, Francis. *Miscellanea curiosa Mathematica, An Introduction to Fluxions, Designed for the Use, and Adapted to the Capacities of Beginners.* Nott's, London, 1777.

Hutton, Charles. *A Mathematical and Philosophical Dictionary, etc.* 2 vols., London, 1795, 1796. New edition, with many additions, etc., 2 vols., London, 1815.

——— *Mathematical Tables . . . Containing . . . Logarithms . . . with Tables Useful in Mathematical Calculations.* To which is prefixed a large . . . history of the . . . writing relating to those subjects, etc. London, 1785.

Jones, William. *Synopsis Palmariorum Matheseos: or, A New Introduction to the Mathematics, etc.* London, 1706.

Ladies' Diary, The. London, Company of Stationers, 1704.

——— *The Diarian Miscellany, Consisting of All the Useful and Entertaining Parts, Both Mathematical and Poetical Extracted from the Ladies' Diary from the Beginning of That Work in the Year 1704 Down to the End of the Year 1773.* With many additional solutions and improvements. In five volumes. By Charles Hutton. Printed for G. Robinson and R. Baldwin, London, 1775.

LANDEN, JOHN. *The Residual Analysis.* London, 1764.

——— *Mathematical Lucubrations; Containing New Improvements in Various Branches of the Mathematics.* London, 1755.

LESLIE, J. "Dissertation Fourth . . . The Progress of Mathematical and Physical Science, Chiefly During the Eighteenth Century," in the seventh edition of the *Encyclopaedia Britannica.*

Lloyd's Evening Post.

L'HÔPITAL (GUILLAUME FRANCOIS ANTOINE DE) MARQUIS DE SAINTE-MESME. *Analyse des infiniment petits, pour l'intelligence des lignes courbes,* 1696. Nouvelle édition, revue et augmentée par Le Fevre. Paris, 1781.

——— *Traité analytique des Sections coniques et de leur usage pour la résolution des équations dans les próblèmes, tant déterminez qu'indéterminez.* Paris, 1720.

——— *An Analytick Treatise of Conic Sections:, . . . made English by E. Stone.* London, 1723.

MACLAURIN, COLIN. *A Treatise of Fluxions in Two Books.* Edinburgh, MDCCXLII.

MASERES, FRANCIS. *The Principles of the Doctrine of Life-Annuities Explained in a Familiar Manner . . . with a Variety of New Tables, of the Values of Such Annuities,* etc. (Appendix. A Bill intituled an Act for the better support of poor persons by enabling Parishes to grant them annuities for life, upon purchase, etc. Directions for using the . . . tables . . . by Dr. R. Price.) London, 1783.

Merlin's Magazine, London, Eighteenth century periodical.

Minutes of the Publick School, 1712–1770, Philadelphia, Pa.

Monthly Review, The, (or Literary Journal). A periodical work giving an account, with proper abstracts of, and extracts from, the new books, pamphlets etc., as they come out. By several hands. 81 volumes. London, (1749–89).

MULLER, JOHN. *A Mathematical Treatise: Containing a System of Conic-Sections; with the Doctrine of Fluxions and Fluents, Applied to Various Subjects.* London, 1736.

——— *A Treatise Containing the Elementary Part of Fortification, Regular and Irregular . . . for the Use of the Royal Academy of Artillery at Woolwich.* Illustrated with 34 copper plates, 1st ed., 1746; 2nd ed., 1756; 3rd ed., 1774.

——— *A Treatise Containing the Practical Part of Fortification, . . . Illustrated . . . For the Use of the Royal Academy at Woolwich.* London, 1755, 3rd ed., 1774.

——— *A Treatise of Artillery, . . . to Which Is Prefixed an Introduction, with a Theory of Powder Applied to Fire Arms.* 2nd ed., with . . . additions. London, 1768, 3rd ed., 1780.

——— *The Attack and Defence of Fortified Places,* in 3 parts, 3rd ed. enlarged, also *Bellidor's New Method of Mining,* and *Vallière on Counter Mines,* 2 pt. London, 1770.

MULLER, JOHN. *The Field Engineer, translated from the French with Observations and Remarks.* 2nd ed. corrected with additions, London, 1773.

——— *Elements of Mathematicks, to Which Is Prefixed the First Principles of Algebra.* 3rd ed improved with *A New Treatise on Perspective.* 2 vols., London, 1765.

——— *A New System of Mathematicks, to Which Is Prefaced the First Principles of Algebra,* etc. London, 1769.

ORESME, NICOLE. *Algorismus proportionum.* From the 15th century MS. in the University of Basel.

——— *Tractatus proportionum.*

——— *Tractatus de uniformitate et difformitate intensionum.*

OUGHTRED, WILLIAM. *Clavis mathematicae.* London, 1631.

Palladium, The. London, Eighteenth century periodical, Robert Heath, editor.

Pennsylvania Gazette, The. Newspaper.

Philosophical Transactions of the Royal Society of London, (1697–1810).

PLAYFAIR, JOHN. *Elements of Geometry, Containing the First Six Books of Euclid.* Edinburgh, 1795.

PRICE, RICHARD. "Treatise on Reversionary Payments." *Philosophical Transactions,* v. lxv, p. 424, 1775.

RAMACHUNDRA, Y. *A Treatise on problems of Maxima and Minima solved by Algebra.* Calcutta, 1850.

RAPHSON, JOSEPH. *History of Fluxions.* London, 1715.

RANDALL, JOHN. *A System of Geography.* London, 1744.

——— *An Account of the Academy at Heath near Wakefield.* London, 1750.

——— *An Introduction to So Much of the Arts and Sciences, More Immediately Concerned in an Excellent Education for Trade, etc.* London, 1765.

——— *A Course of Lectures in the Most Easy . . . and Entertaining Parts of Geography, Astronomy, Chronology, and Pneumatics, etc.* London, 1750.

Records of the Royal Military Academy.

RECORDE, ROBERT. *The Grovnd of Artes.* London, 1540–42.

——— *The Castle of Knowledge.* London, 1551.

——— *The Pathewaie to knowledge.* London, 1551.

——— *The whetstone of witte.* London, 1557.

REISNER, EDWARD HARTMAN. *Nationalism and Education Since 1789. A Social and Political History of Modern Education.* The Macmillan Company, New York and London, 1923.

ROBERTSON, JOHN. *The Elements of Navigation.* London, 1754.

——— *A Compleat Treatise of Mensuration.* London, 1739.

——— *Mathematical Instruments.* London, 1747.

——— Translation of De La Caille's *Elements of Astronomy.* London, 1750.

ROWNING, JOHN. *A Preliminary Discourse to an Intended Treatise on the Fluxionary Method.* London, 1756.

ROWE, JOHN. *An Introduction to the Doctrine of Fluxions. Revised by Several Gentlemen Well Skill'd in the Mathematics.* London, MDCCXLI. Fourth edition, London, 1809, contains a sixteen page essay on *Fluxions* written and published anonymously by Francis Blake.

SACROBOSCO. *Tractatus de Arte Numerandi.* Printed in Halliwell, *Rara Mathematica*, I.

SAUNDERSON, NICHOLAS. *The Elements of Algebra in Ten Books.* Cambridge, 1740.

SIMONS, LAO G. *Introduction of Algebra into the American Schools in the Eighteenth Century.* Bulletin no. 18, Bureau of Education, Superintendent of Documents, Washington, 1924.

SIMPSON, THOMAS. *A New Treatise of Fluxions: Wherein the Direct and Inverse Method Are Demonstrated after a New, Clear and Concise Manner, with Their Applications to Physics and Astronomy.* London, 1737.

——— *The Nature and Laws of Chance.* London, 1740. *To This Work Is Annexed Full and Clear Investigations of Two Important Problems Added in the 2d Edition of Mr. DeMoivre's Book on Chances, as Also Two New Methods for the Summation of Series.*

——— *Essays on Several Curious and Useful Subjects in Speculative and Mixed Mathematicks.* London, 1740.

——— *The Doctrine of Annuities and Reversions: Deduced from General and Evident Principles: with Useful Tables, Shewing the Values of Single and Joint Lives.* London, 1742.

——— *Mathematical Dissertations on a Variety of Physical and Analytical Subjects.* London, 1743.

——— *A Treatise of Algebra; Wherein the Fundamental Principles Are Fully and Clearly Demonstrated, and Applied to the Solution of a Great Variety of Problems, and to a Number of Other Useful Enquiries,* to Which Is Added, *The Construction of a Great Number of Geometrical Problems, with the Method of Resolving Them Numerically.* London, 1745. First American edition reprinted from the eighth, London, was published at Philadelphia for Mathew Carey by T. & G. Palmer, 1809.

——— *The Elements of Geometry, with Their Application to the Mensuration of Superficies and Solids, to the Determination of Maxima and Minima, and to the Construction of a Great Variety of Geometrical Problems.* London, 1747; 2d edition, 1760.

——— *Trigonometry, Plane and Spherical, with the Construction and Application to Logarithms.* London, 1748.

——— *Doctrine and Application of Fluxions: Containing (Besides What Is Common on the Subject), a Number of New Improvements in the Theory, and the Solution of a Variety of New and Very Interesting Problems in Different Branches of the Mathematicks.* London, 1750.

——— *Select Exercises for Young Proficients in the Mathematicks.* London, 1752; ed., 1792.

Simpsoniana—Collection of manuscript material consisting of correspondence and mathematical notes of Thomas Simpson, to be found in the private library of David Eugene Smith, Columbia University.

Simson, Robert. *Sectionum Conicarum Libri V.* Edinburgh, 1735, with an enlarged edition in 1750.

——— *Apollonii Pergaei Locorum Planorum libri II, restituti. . . .* Glasgow, 1749.

——— *Euclid's Elements.* Glasgow, 1756, with numerous editions. Some of these works were published posthumously at Glasgow in 1776.

Smiles, Samuel. *The Huguenots, Their Settlements, Churches, and Industries in England and Ireland.* London, 1895.

Smith, David Eugene. *History of Mathematics.* 2 vols., Ginn and Company, New York, 1925.

Smith, James. *A New Treatise of Fluxions, Containing, I. The Elements of Fluxions, Demonstrated in Two Easy Propositions, Without First or Last Ratios, II. A Treatise of Nascent and Evanescent Quantities, First and Last Ratios, III. Sir Isaac Newton's Demonstration of the Fluxions Enlarged and Illustrated: IV. Answers to the Principal Objections in the Analyst.* London, 1737.

Stone, Edmund. *The Method of Fluxions, Both Direct and Inverse. The Former Being a Translation from the Celebrated Marquis de l'Hospital's Analyse des Infinements Petits: And the Latter Supply'd by the Translator.* London, MDCCXXX.

Sydney, William. *England and the English in the Eighteenth Century.* New York, 1891.

Tartaglia, Nicolo. *Nuova scienza. . . .* Venice, 1537.

——— *General Trattato di nvmeri, et misvre.* 2 parts (volumes), Venice, 1556–1560.

Taylor, Brook. *Linear Perspective.* London, 1715.

——— *Principles of Linear Perspective.* London, 1719.

Truth Triumphant: or, Fluxions for the Ladies. Shewing the Cause to Be Before the Effect, and Different from It; That Space Is Not Speed, Nor Magnitude Motion. With a Philosophic Vision, Most Humbly Dedicated to His Illustrious High and Serene Excellence, the Sun. For the Information of the Public, by $\dot{X}$, $\dot{Y}$, and $\dot{Z}$, Who Are Not of The Family of x, y, and z, but Near Relations of $(\dot{x})$, $\dot{y}$, and $\dot{z}$ . . . London. Printed for W. Owen, MDCCLII.

Turner, John. *Mathematical Exercises.* London, 1750–1752.

——— *The Mathematician, Containing a Dissertation on the Rise, Progress and Improvement of Geometry,* published by a Society of Gentlemen. No. 1–2, London, 1745; no. 1–8, London, 1750.

Tweedie, Charles. *James Stirling, A Sketch of His Life and Writings Along with His Scientific Correspondence.* Oxford Clarendon Press, London, 1922.

Viéte, Francois. *Isagoge in artem analyticam. Effectionum geometricarum canonica recensio,* and *Supplementum Geometriae.* Paris, 1593.

Virginia Gazette, 1772.

VIVIANI, VINCENZO. *De Maximis, et Minimis Geometrica Divinatio in Qvintvm Conicorum Apollonii Pergaei adhvc desideratvm.* Florence, 1659.

WALLINGFORD, RICHARD OF. *Quadripartitum de sinibus demonstratis; De sinibus et arcubus in circulo inveniendo; De chorda et arcu; De chorda et versa.*

WALLIS, JOHN. *Arithmetica Infinitorum, sive Nova Methodus Inquirendi in Curvilineorum Quadraturam, aliaque difficiliora Matheseos Problemata.* Oxford, 1655.

WARD, JOHN. *The Young Mathematician's Guide.* London, 1706.

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